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MAGNETIC PHENOMENA

"The precise derivation of the term 'magnet,' which has now become the most common one, is difficult to ascertain. Lucretius (99–55 B.C.) says it was called 'magnet' from the place from which it was obtained —'in the native hills of the Magnesians.' However, Pliny (23–79 A.D.) relates a prettier legend, as copied from the poet Nicander (second century B.C.), that the shepherd, 'Magnes' by name, while guarding his flock on the slopes of Mount Ida, suddenly found the iron crook of his staff clinging to a 'stone,' which has become known after him as the 'Magnes stone' or magnet."

—BAUER, "Principal Facts of the Earth's Magnetism."

MAGNETIC PHENOMENA

AN ELEMENTARY TREATISE

BY

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TO MY WIFE

WHOSE HELP AND LOYALTY MADE
THIS BOOK POSSIBLE

FOREWORD

The experience of twenty years of undergraduate teaching has shown that college students may be successfully introduced to research work. If the rising generation is to play a leading part in the program of scientific research, much more attention must be paid in the future to arousing the interest of undergraduates in the various fields of science and to inspiring them with the spirit of research. The professions which college men and women follow when they leave college are to a large extent determined by the kind of work that has stimulated them in their undergraduate days.

All will agree that the greatest stimulus a student may have for productive scholarship is a teacher wide awake and enthusiastic for his subject, inspiring his students to ask all sorts of questions about the problem in hand. Such a teacher must himself be filled with a desire to extend his own field of knowledge by study and research. One engaged in research will, in general, make a better teacher and, on the other hand, teaching reacts favorably on the research work. The next great stimulus to scholarship is a good book. Many a student owes his inspiration to the suggestions offered by a book which he has chanced to read.

Naturally, the embryo researcher is often found asking the question; "What and how shall I investigate?" He must needs be introduced to those fields which are so alluring to the older investigator and be led to select some subject which will appeal to him. Not only must he be oriented in a given subject but he must become acquainted with the various types of technique necessary to do the work. It is here that the successful teacher or book finds his or its opportunity.

While the subject of *magnetism* is a very old one, it still remains the least understood and also the most neglected of subjects. The investigators who have devoted their lives to this field are few in number. The great trouble with the research work in magnetism is that too many indulge themselves in some problem for a year or two and then drop it for work in other fields. They have developed no background for magnetic

investigations and are, therefore, not able to see the specific problem in its relations to the general field.

This book is written in the hope that some youthful adventurer in the field of physics may be induced to choose some phase of the subject of magnetism as a life interest. It is desired that in the course of the succeeding chapters it may be pointed out how many problems are urgently calling for a solution. There is a hope that a beginning in the methods and technique of such investigations may be disclosed as the various subjects are presented, and the novitiate led to try some of the problems himself. Perhaps this volume may also be of help to other teachers who, with the author, are seeking to bridge successfully the gap between undergraduate and graduate days and, at the same time, trying to help young people find themselves in their life work.

Many undergraduate laboratories of physics are not supplied with the most up-to-date apparatus. Nevertheless, a great deal of valuable research work in magnetism can be done with comparatively simple and inexpensive apparatus. It is not a question of "*What could I do if I had the equipment?*" but "*What can I do with the equipment at my disposal?*"

These pages are intended as a guide to the beginner rather than an exhaustive treatise on magnetic phenomena. The references, therefore, are confined to those which, in the judgment of the writer, are the most leading and comprehensive. References not specifically noted in this book will generally be found in those articles which are cited. This will enable the student to make as thorough a study of the subject as he may desire.

The goal of this book then is to stimulate in upper classmen in college the spirit of research, and possibly enlist the interest of those who have just finished their graduate work and are casting about for a field of investigation which may be inaugurated on their own initiative.

I have endeavoured every where to make myself as intelligible as possible, and have omitted nothing, that appeared likely to prevent any mistakes. This may make some things perhaps seem tedious, or unnecessarily repeated; but if any person thinks so, I must beg him to consider, that all people are not equally ready at comprehending descriptions of things they were not before acquainted with; and that, therefore, he ought to bear a little with what to him may seem unnecessary, for the sake of others less acute and ingenious than himself.[1]

[1] MICHELL, "Artificial Magnets," 2d ed., p. 27, 1751.

Acknowledgments

I am under great obligations to all the authors referred to in this book. In many cases very lavish use has been made of suggestions regarding order of presentation, figures, and terminology.

I am indebted to the National Research Council for permission to use some of the material published in "The International Critical Tables" given in the section on Magnetic Data.

Charles Scribner's Sons kindly permitted the use of the picture in the frontispiece; Maschinenfabrik Oerlikon furnished the excellent photographs of various electromagnets; the director of the Hydrographic Office of the United States Navy kindly allowed the use of Declination Chart 2406 and Inclination Chart 1700; Mr. Frank Fahy furnished photographs of his permeameter; colleagues and friends helped in various discussions of subject matter; Messrs. Sidney Barnes and Paul Mitchell assisted in the preparation of the illustrations.

To all of these contributors I extend my most cordial thanks.

S. R. Williams.

Amherst, Mass.,
September, 1930.

CONTENTS

INTRODUCTION

A MAGNETIC STUDY OF THE PHYSICAL AND CHEMICAL PROPERTIES OF MATTER

When a substance is exposed to the influence of a magnetic field, it behaves in various ways, depending upon the physical and chemical properties of the material examined. Oxygen is attracted to the poles of a magnet while carbon dioxide is repelled. Bismuth shows a marked change in resistance when magnetized, while copper shows relatively little. Steels of varying degrees of hardness indicate corresponding variations in the changes of length produced by a magnetic field. Each substance, whether it be a gas, a liquid, or a solid, discloses its own peculiar characteristics when magnetized.

Magnetic phenomena are classified as "effects" according to the behavior which matter is observed to undergo when magnetized. If a magnetic field changes the optical properties of a substance it is called a magneto-optical effect. This is a very suggestive term. Unfortunately, corresponding terms to designate those effects which are produced when a magnetic field changes the mechanical, acoustical, electrical, magnetical, and thermal properties of matter have not been adopted to any great extent. While it may be unorthodox, nevertheless, such a division gives an excellent bird's-eye view of the subject matter in magnetic phenomena. Introducing these terms which correspond to the term magneto-optical, the following outline of magnetic phenomena indicates for the most part the topics to be discussed in successive chapters.

OUTLINE OF MAGNETIC PHENOMENA

I. Magneto-magnetics.
   1. The magnetic field.
   2. Forces in dia-, para-, and ferromagnetic media.
   3. Magnetic induction.
   4. Intensity.
   5. Hysteresis.
   6. Permeability.

7. Susceptibility.
8. Coercive force.
9. Retentivity.
10. Reluctance.
11. Leakage.

II. Magneto-mechanics.
1. Mechanical strains due to magnetic stresses:
   a. Linear changes;
   b. Circular changes;
   c. Volume changes.
2. Magnetic strains due to mechanical stresses:
   a. Linear changes;
   b. Circular changes;
   c. Volume changes.
3. Mechanical effects of one magnetic field on another.

III. Magneto-acoustics.
1. Production of sound by magnetization.
2. Influence of magnetism on periodicity.
3. Influence of vibrations on magnetism.

IV. Magneto-electrics.
1. Electro-magnetic phenomena.
2. Hall effect.
3. Ettingshausen effect.
4. Change in resistance due to a magnetic field.
5. Galvanomagnetic longitudinal temperature difference.
6. EMF due to magnetized condition.
7. EMF of a voltaic cell varied by a magnetic field.
8. Change in thermo-electric properties.

V. Magneto-thermics.
1. Nernst effect.
2. Righi-Leduc effect.
3. Thermomagnetic longitudinal potential difference.
4. Thermomagnetic longitudinal temperature difference.
5. Influence of heat on magnetic phenomena.
6. Energy dissipation and development of heat in hysteresis.
7. Pyro- and piezomagnetism.
8. Change in boiling point and specific heat due to a magnetic field.

VI. Magneto-optics.
1. Faraday effect.
2. Kerr effect.
3. Zeemann effects.
4. Magnetic double refraction.
5. Effect of light on magnetism.

VII. Cosmical Magnetism.

Keeping in mind what has been said in regard to magneto-optics, the field of magneto-mechanics may be said to include those magnetic phenomena in which mechanical effects occur due to a magnetic field. Change in dimensions produced by a

magnetizing force is an illustration of this type of phenomena. Also, those reciprocal effects in which mechanical stresses produce changes in the magnetic properties come under this head. The terms serving as captions for the other subdivisions may be defined in a similar fashion. For instance, the field of magneto-acoustics may be taken to cover those phenomena in which sound is produced by a magnetic field as well as those changes in magnetic properties due to sound vibrations.

If all substances could be examined with reference to the effects classified in the first six divisions of the outline, it would be not only a complete magnetic analysis of all of the physical properties but a tabulation of their chemical properties as well. In magneto-optics, for example, if the Faraday effect were studied in all substances, our knowledge would not be complete until the chemical relations were all known. Even after a complete magnetic survey of matter was made, the study would not be ended for there are all sorts of correlations to be made between the six groups of magnetic phenomena. For example, in magneto-acoustics it is observed that the period of a tuning-fork is varied by being magnetized. In magneto-mechanics there is the well-known effect that magnetism changes the dimensions of steel and also its moduli of elasticity. Here are several phenomena about steel which may be correlated. To the extent that physical phenomena may be correlated and coordinated, to that degree is progress made. As another illustration of correlation it seems to have been rather conclusively established in the field of magneto-mechanics[1] that there is one and only one set of mechanical characteristics corresponding to a given set of magnetic characteristics and, *vice versa*, there is one and only one set of magnetic characteristics corresponding to a given set of mechanical characteristics. Can this principle, which is so important in magnetostriction, for example, be applied to the other five fields of magnetic phenomena? If this is possible then the generalized statement may be expressed as follows:

There is one and only one set of physical and chemical properties corresponding to a given set of magnetic characteristics, and, conversely, there is one and only one set of magnetic properties corresponding to a given set of physical and chemical characteristics.

This principle becomes a program of research that must challenge many, because of the unlimited opportunities it gives for

[1] Burrows, *Bull.* Bur. Stand., **13**, 207, 1916.

research problems. It suggests vital relations between magnetism and atomic theories and points the way to valuable analytical methods in the industrial arts.

In attempting to stimulate research in the field of magnetism there are several points which should be emphasized.

1. As far as possible the same specimens should be used in all of the magnetic tests which may be applied to any substance. For instance, if the magneto-mechanical effects were studied in a particular steel rod, that same rod should be used for studying the Barkhausen effect (magneto-acoustics), the change in resistance (magneto-electrics), its magnetic induction (magneto-magnetics), the effects due to heat (magneto-thermics), and the Kerr effect (magneto-optics). *The importance of making as many tests as possible on the same specimens cannot be overemphasized.*

2. Here, if anywhere, in scientific research, there must be cooperative action, for the program of research, as outlined above, will lead into the fields of physics, chemistry, both physical and chemical metallurgy, and mineralogy. A group of physicists, mineralogists expert in x-ray analysis, chemists, and metallurgists cooperating in the study of magnetic phenomena could make a wonderful contribution to this field. Helmholtz is reported as having said, "the disgrace of the nineteenth century is its ignorance of the subject of magnetism." Will the twentieth bring knowledge?

3. In spite of what has been said in a preceding paragraph concerning the neglect of the study of magnetism, there has been a prodigious amount of labor performed in the field of magnetic research. Knowlton[1] counted the papers on the subject of magnetism which had been reviewed in *Science Abstracts* for the decade, 1900–1910. There were about 500, exclusive of technical contributions and papers on electromagnetism. *Sustained research, both coordinated and cooperative, is needed in the fields of magnetic phenomena.*

4. Much work has been done on materials whose previous chemical and physical histories were not known. Both of these are highly important in correlative studies. As a result of this kind of research, magnetic phenomena lack a unifying theory. There is real need to go back and review the experimental data and, where possible, correlate one set of facts with

[1] KNOWLTON, *Terr. Mag. Atmos. Elec.*, **15**, 3, 1910.

another. Where no knowledge of the physical and chemical history exists the experiments should be repeated as far as possible with standard or known conditions. *Modern methods of pyrometry, microphotography, chemical and* x-*ray analysis give no excuse for ignorance of the character of the materials used in magnetic investigations.*

5. The magnetic analysis of the physical and chemical properties of matter will lead into many fields that at present are almost wholly unexplored. This is illustrated in that work which has been undertaken rather recently, *viz.*, the study of the mechanical properties of steel and other ferromagnetic substances by their behavior in a magnetic field. In the case of diamagnetic material the magnetic forces are so feeble that they may be investigated only by very delicate methods. *This difficulty will disappear as the technique of the problem is developed.*[1]

6. Finally, it is exceedingly important for the beginner to know the technique of finding out what has already been done in the field of knowledge in which he is interested. This consists chiefly in a mechanical thumbing of the indices of books, with much reading, and in finding the topics which might bear on the subject. There are, however, certain books which, all will agree, are useful adjuncts to research and should be on the shelves of any library where bibliographic work in physics is expected to be done. Among these may be named:

A good dictionary.
Annalen der Physik, Beiblätter, Abstract Section.
CHWOLSON, "Traité de Physique."
Encyclopedias (Britannica).
GEIGER and SHEEL, "Handbuch der Physik."
GLAZEBROOK, "Dictionary of Physics."
"International Catalog of Scientific Literature."
"International Critical Tables."
Japanese Journal of Physics, Abstract Section.
LAMONT, "Handbuch des Magnetismus," "Allgemeine Encyclopädie der Physik," Vol. 15, 1867.
LANDOLT-BORNSTEIN-ROTH and SCHEEL, "Physikalisch-Chemische Tabellen."
"Revue bibliographique," *Journal de physique,* Abstract Section.
"Royal Society Catalog of Scientific Papers."
Science Abstracts.
WIEDEMANN, "Lehre von der Elektricität."
WIEN and HARMS, "Handbuch der Physik."
WINKELMANN, "Handbuch der Physik."

[1] HONDA and ENDO, *Proc.* Inst. Metals, Gen. Meet., Mar. 10, 1927.

About such a group of books should be built a working library of the best scientific periodicals, both current and former issues. A list of these periodicals would include the following:

Annalen der Physik.
Astrophysical Journal.
Comptes rendus.
Indian Journal of Physics.
Japanese Journal of Physics.
Journal de physique.
Journal of the Optical Society of America.
Journal of the Franklin Institute.
Nature.
Nuovo Cimento.
Philosophical Magazine, London, Edinburgh, and Dublin.
Physical Review.
Physikalische Zeitschrift.
Science.
Science Progress.
Terrestrial Magnetism and Atmospheric Electricity.
Zeitschrift für Physik.

To these should be added as many *Proceedings* and *Transactions* of scientific societies as are within the means of the library.

Books, wholly or in part on the subject of magnetism, which the author has consulted frequently and to which he wishes to express his obligations in this work are the following:

Airy, "A Treatise on Magnetism."
Andrade, "The Structure of the Atom."
Auerbach, "Modern Magnetics," transl. by Booth.
Back and Lande, "Zeemanneffekt und Multiplettstruktur der Spectrallinien."
Bauer, "Principal Facts of the Earth's Magnetism."
Campbell, "Modern Electrical Theory";
"The Structure of the Atom";
"Galvanomagnetic and Thermomagnetic Effects."
Chree, "Studies in Terrestrial Magnetism."
duBois, "The Magnetic Circuit in Theory and Practice."
Ebert, "Magnetische Kraftfelder."
Emtage, "Electricity and Magnetism."
Ewing, "Magnetic Induction in Iron and other Metals," 3d ed.
Faraday, "Experimental Researches," 3 Vols.
Foster and Porter, "Electricity and Magnetism."
Franklin and MacNutt, "Advanced Electricity and Magnetism."
Gans, "Einführung in die Theorie des Magnetismus."
Gauss, "Allgemeine Theorie des Erdmagnetismus."
Gilbert, "De Magnete," transl. by Gilbert Club.
Gilbert, A. C., "Magnetic Fun and Fact."

GLAZEBROOK, "Electricity and Magnetism."
GORDON, "Electricity and Magnetism," 2 Vols.
GRAETZ, "Handbuch der Elektricität und Magnetismus."
GRAY, "A Treatise on Magnetism and Electricity."
HADLEY, "Magnetism and Electricity for Students."
HARRIS, "Rudimentary Magnetism."
HAZARD, "Directions for Magnetic Measurements."
HONDA, "Magnetic Properties of Matter."
HOUSTON and KENNELLY, "Magnetism."
HUTCHINSON, "Advanced Textbook of Magnetism and Electricity," 2 Vols.
JEANS, "Electricity and Magnetism."
KARAPETOFF, "The Magnetic Circuit."
LAMB, "Notes on Magnetism."
LAMONT, "Erdmagnetismus";
"Handbuch des Magnetismus," "Allgemeine Encyclopädie der Physik," Vol. 15.
LEWIS, "The Effects of a Magnetic Field on Radiation."
LLOYD, "A Treatise on Magnetism."
MANSFIELD, "Electromagnets, Their Design and Construction."
MASCART and JOUBERT, "Electricity and Magnetism," transl. by Atkinson.
MATTEUCCI, "Magnetisme de Rotation."
MAURAIN, "Le Magnetisme du Fer."
MAXWELL, "Electricity and Magnetism," 2 Vols.
METCALF, "A New Theory of Terrestrial Magnetism."
MICHELL, "Artificial Magnets."
National Research Council *Report*, No. 18, "Theories of Magnetism."
NECULCEA, "La Phénomène de Kerr et les Phénomènes Electro-optiques."
NIPHER, "Electricity and Magnetism";
"Theory of Magnetic Measurements."
NIPPOLDT, "Erdmagnetismus, Erdstrom und Polarlicht."
PEDDIE, "Molecular Magnetism."
PERKINS, "Electricity and Magnetism."
PIDDUCK, "A Treatise on Electricity," 2d ed.
POYNTING and THOMSON, "Electricity and Magnetism."
RADAU, "Le Magnetisme."
RICHARDSON, "The Electron Theory of Matter."
SCORESBY, "Magnetical Investigations," 2 Vols.
SMITH, "Electrical and Magnetic Measurements."
SOMMERFELD, "Atomic Structure and Spectral Lines."
SPOONER, "Properties and Testing of Magnetic Materials."
STARKE, "Electricität und Magnetismus."
STARLING, "Electricity and Magnetism."
STONER, "Magnetism and Atomic Structure."
THOMPSON, "Lectures on the Electromagnet";
"The Electromagnet."
THOMSON, "Elements of Electricity and Magnetism";
"Application of Dynamics to Physics and Chemistry";
"Papers on Electrostatics and Magnetism."
TIMBIE and BUSH, "Principles of Electrical Engineering."

TYNDALL, "Diamagnetic and Magnecrystallic Action."
UNDERHILL, "Magnets";
"Solenoids."
VOIGT, "Magneto-Optik" in GRAETZ, "Handbuch der Elektricität und Magnetismus."
WALKER, "On the Magnetism of Ships and the Mariner's Compass";
"Terrestrial and Cosmical Magnetism."
WALL, "Applied Magnetism."
WEDEKIND, "Magnetochemie."
WOOD, "Physical Optics," chaps. on Magneto-optics.
ZEEMANN, "Magneto-optics."

The outstanding problems and phases of research in the field of magnetic phenomena may be summarized as follows:

1. An improvement in all forms of instruments for magnetic measurements. This in itself constitutes an important field of research.

2. Repetition of a great deal of the work which has already been done, using improved methods and materials which may be duplicated very closely by others.

3. Correlation of various magnetic phenomena by data obtained from the same specimens. The importance of this cannot be overestimated.

4. Cooperative research. A group of physicists, mineralogists expert in x-ray analysis, chemists, and metallurgists, cooperating in the study of magnetic phenomena, could make a wonderful contribution to this field.

5. Sustained research. Volunteers for life in the field of magnetic research are essential to definite advances.

6. Better summaries and digests of the literature dealing with experimental studies of magnetic phenomena. Good bibliographies, so that the investigator may find as expeditiously as possible what has already been done in the field of magnetic phenomena.

A general study, as thus outlined, would lead to two very distinct goals: first, it would develop *methods of analysis*, of immense value to the industrial arts and sciences; and secondly, of even greater significance, a furtherance of our present *concept of the structure of the atom.*

MAGNETIC PHENOMENA

CHAPTER I

MAGNETO-MAGNETICS

1. The Magnetic Field.—As early as 600 B.C. it was observed that the natural mineral, magnetite ($FeO.Fe_3O_4$),[1] possessed the property of attracting small bits of iron to itself (Fig. 1). A piece of magnetite is called a natural magnet in contradistinction to artificial magnets which may be made by rubbing[2] a piece of steel, such as a knitting needle, with a piece of lodestone or magnetite. A substance is said to be magnetized when it possesses the property of attracting to itself small bits of iron as does magnetite. About either the magnetite or the magnetized steel needle is a space in which this power to attract small particles of iron may be observed by proper means. We speak of such a space as a *magnetic field of force*. Theoretically it extends to infinity but the detectable field of an ordinary bar magnet does not reach many meters away from the bar.

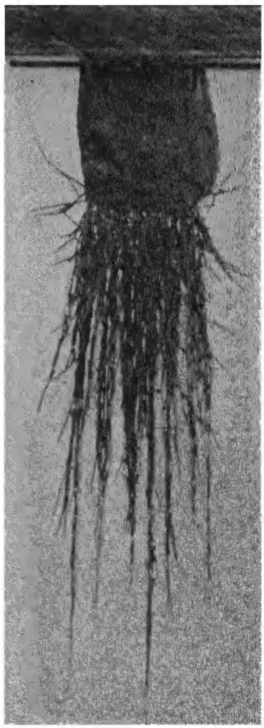


FIG. 1.—The lodestone attracts small pieces of iron. The small bits of iron are short sections of fine iron wire.

In Figs. 2 and 3 these fields of force have been visualized by sprinkling iron filings over a magnet lying on a photographic plate. A gentle tapping of the plate causes the filings to arrange

[1] WELO and BAUDISCH, *Philos. Mag.*, **3**, 396, 1927.

[2] POYNTING and THOMSON, "Electricity and Magnetism," pts. 1 and 2, p. 70, 1920.

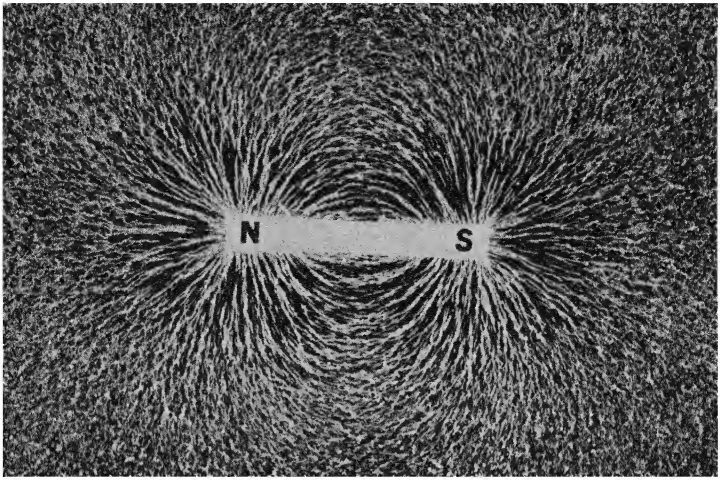


FIG. 2.—The magnetic field about a bar magnet. Field is portrayed by scattering iron filings about the magnet.

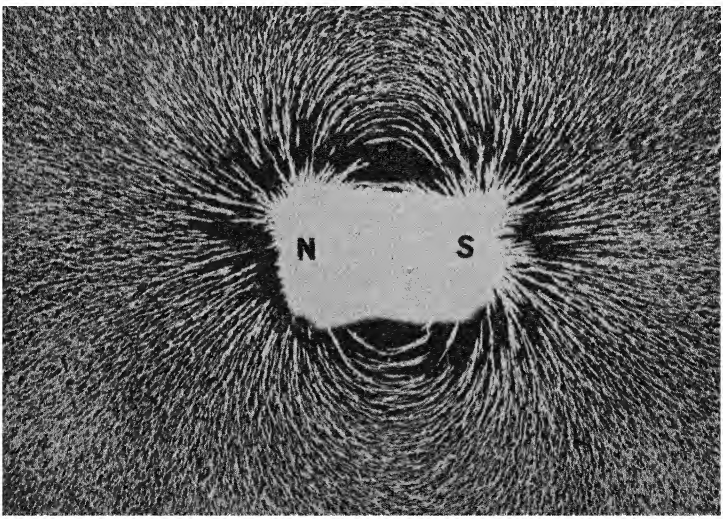


FIG. 3.—The magnetic field about a lodestone. The iron filings align themselves along the lines of force.

themselves in chains about the magnet. These lines of filings indicate the paths along which the magnetic force acts. To get a permanent picture of the magnet and filings, the plate is exposed to light and then developed. Since all magnetic phenomena are due to a magnetic field it is important to have the terms in mind whereby such a field of force is described.

2. Magnetic Poles.—In scattering the filings about a magnet it will be observed that not only are they attracted by the iron but they arrange themselves along definite lines. These lines appear to begin and end in two fairly distinct regions, N and S, which are called the *poles* of the magnet. Conventionally, the two regions N and S are designated as the positive and negative poles of the magnet respectively. Actually, a magnetic pole is not a mathematical point. In practice, however, the position of the magnetic pole may be taken as that point toward which all the lines of the filings seem to converge. The line connecting these two points in N and S is called the *axis* of the magnet. A piece of magnetite or a steel magnet, when suspended freely in a horizontal position by a fiber, will set its axis north and south in the earth's magnetic field. That pole which turns toward the north is called the north-seeking pole (positive), and the one swinging toward the south is the south-seeking pole (negative). For very long, slim magnets it may be assumed, without great error, that the poles are situated at the ends of the magnet. This is far from being true in the case of short magnets. A permanently magnetized steel ball has its poles quite near the center of the sphere. Nevertheless, there are two distinct centers or points from which the magnetic forces seem to operate (Fig. 4).

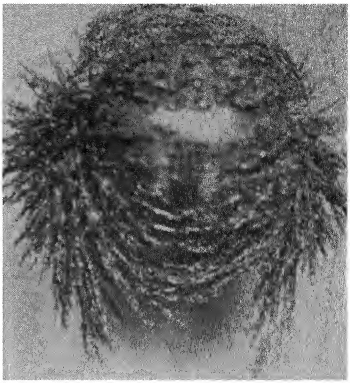


FIG. 4.—Magnetized steel ball. Definite poles are indicated.

In the process of magnetizing a piece of steel it may occur that a north pole is formed at each end and two south poles in the middle, or *vice versa*.[1] Such a combination is spoken of as a

[1] POYNTING and THOMSON, "Electricity and Magnetism," p. 174, 1920.

magnet with *consequent poles.* Figures 5 and 6 indicate such conditions. Figure 6 shows a rather unusual case.

From the standpoint of magnetic theory the difference between electric charges and magnetic poles should be emphasized (see p. 124). In electric phenomena it is possible to separate electric charges; in fact, the process of electrification is nothing more than the separation of positive and negative charges. The process of magnetization is quite different because positive and

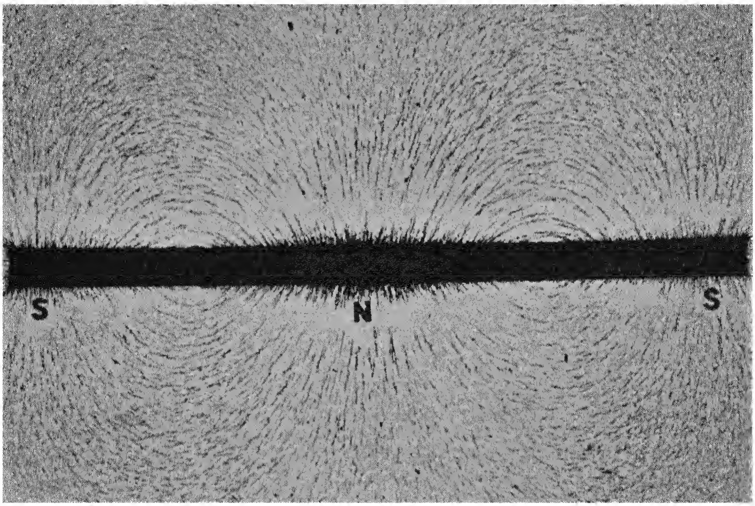


FIG. 5.—Consequent poles in a slim bar magnet. Like poles at the two ends.

negative poles cannot be separated. Break a magnetized knitting needle in two. One does not hold in one hand a positive pole and in the other a negative; rather, two magnets are formed. In turn, each piece may be broken in two and again two magnets are formed. One can conceive of this process going on until there comes a division in which two magnets are not the end result. *What is that ultimate particle which does not yield two magnets when broken in two?* The answer to that question is one of the most important in the field of magnetism today. It should be in the background of every problem of research in this field. For the present let us call that ultimate particle the *Magneton.*

An electric current[1] always produces a magnetic field. This leads us to believe that the magneton is due to atomic currents, *i.e.*, electrons revolving about nuclei or perhaps spinning on their own axes. If the magnetons, are atomic currents then it may be said of magnetic poles in general that they occur only where the distribution of electric circuits becomes non-uniform. This idea may be extended to the magnetic poles associated with a solenoid. The poles will be found near the ends of the coil,

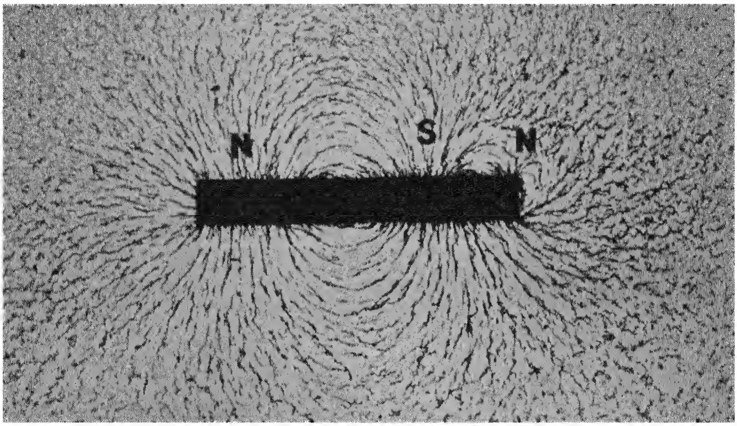


FIG. 6.—Peculiar position of consequent poles in a small bar magnet.

i.e., where the distribution of electric circuits is non-uniform. Magnetic poles are not essential to magnetic fields. This is shown in the case of the field of a toroidal electromagnet.

The concept of magnetic poles seems to have arisen because of convenience in treating certain problems connected with magnetic fields. These concepts[2] do not appear so important today in the light of our modern theories of magnetism. Warburton[3] has recently raised the question, "Will the magnetic pole vanish?" Furthermore, it must be kept clearly in mind that an isolated magnetic pole is not attainable experimentally.

[1] OERSTED, *Ann. Philos.*, **16**, 273, 1820;
ROWLAND, *Philos. Mag.*, **27**, 445, 1889;
TOLMAN and McRAE, *Phys. Rev.*, **34**, 1075, 1929.

[2] KARAPETOFF, "The Magnetic Circuit," see note p. 263.

[3] WARBURTON, *Prog.*, Amer. Phys. Soc., Des Moines, Iowa, December., 1929.

3. Lines of Force.—When iron filings are scattered around a steel magnet, as in Fig. 2 or between the like or unlike poles of two magnets, as in Figs. 7 and 8, the filings arrange themselves in definite lines or filaments. This is a graphical study of a magnetic field surrounding a magnet. Seeing these lines of filings undoubtedly led Euler, in writing to a German princess in 1761, to describe them as "lines of flow of the magnetic fluid." This idea Faraday emphasized in his writings and utilized as a method for describing the paths along which the magnetic forces acted.

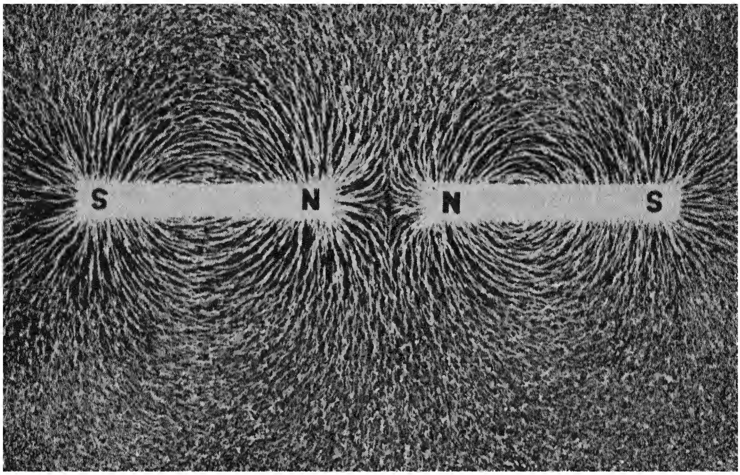


FIG. 7.—The magnetic field surrounding two small bar magnets. Two like poles oppose each other at ends nearest each other.

Faraday called them *lines of force.* In his mind's eye, Faraday saw these lines of force bristling from every magnet whether there were iron filings there to indicate them or not. The story is told that he was accustomed to play with a small compass during worship and picture the lines of force which surrounded the needle. There is no physical existence of these lines of force. They are simply a convenient means for describing a magnetic field. In the case of electric charges it is always stated that the electric lines of force begin on a positive charge and end on a negative charge. The case is different for magnetic lines of force. While they appear to pass from one pole to another, in

reality the magnetic lines of force are closed lines as indicated in Fig. 9. The evidence for this statement will appear later in discussing the magnetic field about a coil of wire carrying an electric current.

Not a little confusion has crept into the literature on magnetism by using the terms "lines of force" and "tubes of force" indiscriminately. It is in the use of the word "line" that the trouble begins. We draw lines with ink or lead about a magnet and call them lines of force. It is not until quantitative significance is

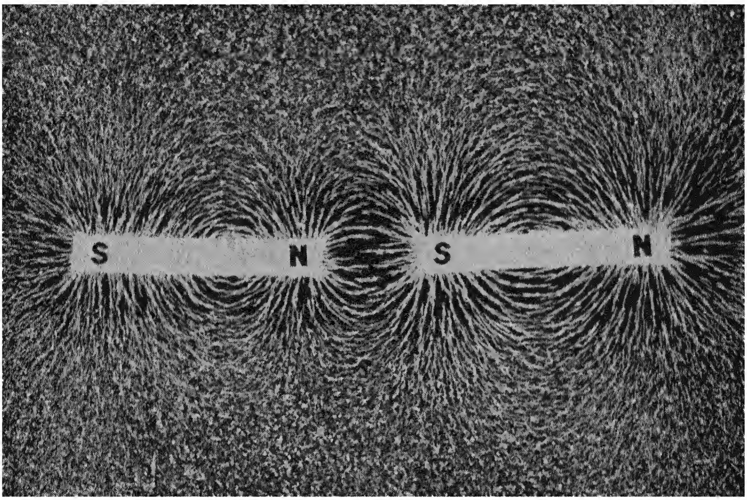


Fig. 8.—The magnetic field surrounding two small bar magnets. Two unlike poles attract each other at ends nearest each other.

given to the term "line of force" that the confusion disappears. In 1831 Faraday[1] made the epochal discovery that when an electrical conductor cuts magnetic lines of force, a potential difference is developed between the ends of the conductor. To illustrate what is meant by the expression, "cutting the lines of force," reference is made to Fig. 10. The rectangle $ABCD$ may represent the face of the north pole of a magnet from which issue a certain number of lines of force called the total *magnetic flux*. N, the number of lines of force per unit area, is called the *flux density*. The conductor LM, as it moves to

[1] Faraday, "Experimental Researches in Electricity," vol. 1, p. 11, 1839.

the right along the rails RR, cuts the magnetic lines emerging from the north magnetic pole. If E is the length of LM included between the rails, and the velocity is v cm. per second, then NEv

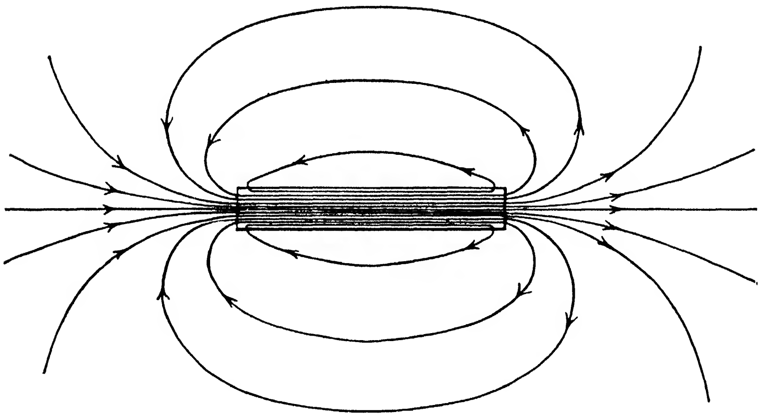


FIG. 9.—Magnetic lines of force are closed lines of force.

equals the number of lines of force cut per second. If the number of lines of force cut per second by a conductor develops an absolute unit of potential difference between its ends, then, by

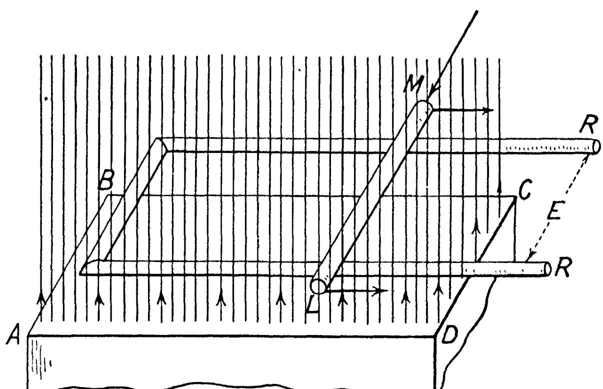


FIG. 10.—As the conductor LM moves to the right along the rails RR, a certain number of lines of force will be cut, or the number of lines of force linked with the conductor will be varied. The rate of cutting or the rate of change of lines linked with conductor determines the electromotive force developed.

definition, the total of the magnetic lines of force cut per second is called a *maxwell of flux*. The maxwell is therefore the unit of magnetic flux. The volt is equal to 10^8 absolute units of potential

difference. It follows, therefore, that a conductor must cut 10^8 maxwells of magnetic flux per second in order that a potential difference of one volt may exist between the ends of the conductor. In Fig. 10 the electromotive force in volts, developed by the motion of the conductor, will be,

$$\text{Emf} = \frac{NEv}{10^8} \text{ volts.} \tag{1}$$

Let us return once more to the concept of "line of force." To represent the field about a bar magnet, an indefinite number of lines may be drawn with pen or pencil. One could and does call these lines, "lines of force." If we should think of a tube inclosing enough of these lines of force so that the cutting of them by a conductor in a second of time would develop a potential difference of one absolute unit, then the group of lines so inclosed would be called a "tube of force." Such a *tube of force* could be and is called by some authors a line of force. The first kind of line of force which was drawn could be indefinite in number, but the tube of force just defined can have only a definite number for any given magnet.[1] If one draws lines of force about a magnet to represent tubes of force it must be clearly stated that such is the case. Similarly, lines may be drawn to represent tubes of force containing 10^8 maxwells of flux; 10^8 maxwells of flux are sometimes called a *weber of flux*. To avoid confusion the kind of lines of force must always be specifically given.

4. Forces between Magnetic Poles.—Coulomb,[2] experimenting with the forces which act between magnetic poles, formulated two laws governing their behavior:

1. Like magnetic poles repel each other, and unlike poles attract each other.

2. The magnitudes of these forces of repulsion and attraction are proportional to the products of the pole strengths and inversely proportional to the square of the distances between them.

$$F = \frac{1}{\mu}\frac{m_1 m_2}{d^2}, \tag{2}$$

where $1/\mu$ is a constant of proportionality. This constant has a physical significance, however, which was emphasized in the researches of Faraday. Before his time the forces between

[1] Gauss' law, sec. **7**.

[2] Coulomb, *Mém. de l'acad.*, Paris, p. 593, 1785.

magnetic poles were considered from the standpoint of "action at a distance." The medium between the poles did not play any part in the calculations. One of Faraday's great contributions was to show that the forces between magnetic poles were influenced by the character of the medium between them. Faraday thought of his lines of force as passing through some media more readily than others, *i.e.*, some substances were more permeable than others to the magnetic lines of force. This factor was taken care of by the constant μ in Eq. (2). Conventionally, μ was taken as equal to unity when the poles were placed in a vacuum or air. If μ was greater than unity, then the force between the poles was not as great as when μ was unity or less. μ is called the factor of permeability. According to its value all substances may be arranged in three classes. If μ is less than unity in any substance it is called a *diamagnetic* body. When a substance shows a value of μ greater than unity it is called a *paramagnetic* material. For most of the substances in the world, μ is a constant no matter what the intensity of the magnetic field may be. There are a few substances in which this is not true and they are always paramagnetic bodies. Iron, nickel, cobalt, manganese under certain conditions, and certain alloys of these metals constitute the major part of this class of bodies, known as *ferromagnetics.*

The best physical conception of the factor μ is obtained from a consideration of Faraday's lines of force threading through a substance. If N is the number of lines of force per square centimeter in a vacuum and if N' is the number through the same area when it is filled with another substance, then $N'/N = \mu$, the same constant of permeability which appears in Eq. (2). For a vacuum, or practically for air, μ is equal to unity. N and N' are physically the same. Making μ have dimensions has always seemed to the author a wrong conception.[1]

For *diamagnetic* bodies N is greater than N'. The lines of force seem to avoid the body and go around it as shown in Fig. 11. For *paramagnetic* bodies just the reverse holds, and N' is greater than N. A paramagnetic body seems to gather in the magnetic lines of force as shown in Fig. 12. All this means that a diamagnetic body will tend to move from a stronger part to weaker parts of a magnetic field, while the paramagnetic substance will do just the opposite. For this reason a paramagnetic needle will

[1] KARAPETOFF, "The Magnetic Circuit," pp. 262–267 (see table II).

set itself parallel to a non-uniform magnetic field while a diamagnetic needle turns with axis normal to the field.[1] Liquids will assume surfaces other than level when in a magnetic field. Figure 13 shows a diamagnetic liquid seeking to move so as to make the least possible obstruction to the magnetic lines of force. Figure 14 shows a paramagnetic liquid gathering itself together

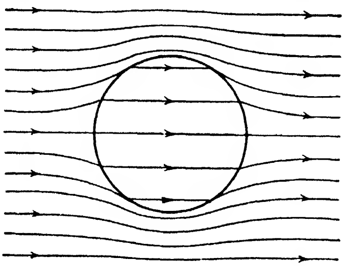


FIG. 11.—Diamagnetic bodies are less permeable to magnetic lines of force than to air or a vacuum.

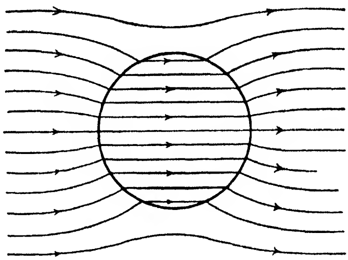


FIG. 12.—Paramagnetic bodies are more permeable to magnetic lines of force than to air or a vacuum.

so that as many lines as possible will go through it. As a general principle of dynamics any body, free to move in a magnetic field, will so move that the potential energy of the system will reduce to a minimum.[2] It is on the basis of this principle that the free movement of any body in a magnetic field may be explained.

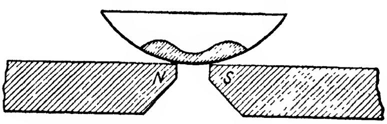


FIG. 13.—A diamagnetic liquid moves in the glass dish so that it will offer a minimum of obstruction to the lines of force.

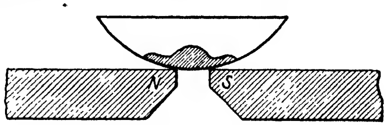


FIG. 14.—Paramagnetic liquids will move in the glass dish so that the greatest depth will be at points of greatest field intensities.

Lord Kelvin[3] has shown that if the applied field were not affected by the induced magnetization, there would be a tendency on the part of both paramagnetic and diamagnetic needles to take up a position parallel to the field. Inasmuch as these two varieties of substances do affect the impressed field differently, the motion is always such that a diamagnetic needle will turn

[1] POYNTING and THOMSON, "Electricity and Magnetism," p. 205, 1920.
[2] STARKE, "Experimentelle Elektricitätslehre," 2d ed., p. 77, 1910.
[3] THOMSON, "Papers on Electrostatics and Magnetism," sect. 691; POYNTING and THOMSON, "Electricity and Magnetism," p. 258, 1920.

normal to the field and a paramagnetic rod parallel. In speaking of these two classes of substances it is understood, of course, that the third class, ferromagnetics, is included under paramagnetics. Strictly speaking a *ferromagnetic* body is distinguished by the fact that μ varies otherwise than linearly with the field applied.

5. Unit Magnetic Pole—Pole Strength.—Equation (2) is an expression for the second law of Coulomb. At the same time it is the defining equation for a unit magnetic pole. This is indicated by making all of the quantities in (2) equal to unity. *A unit magnetic pole will exert a force of one dyne on a similar pole when separated by a distance of one centimeter in a vacuum or air.* The strength of any magnetic pole, therefore, is the force in dynes which that pole exerts on a unit magnetic pole when placed at a distance of one centimeter from it in a vacuum or air.

Even though the idea of magnetic poles may have lost its importance, it is to be noted that it is still deeply rooted in our definitions and concepts. The term magnetic pole will not disappear from our literature immediately.

6. Magnetic Field Strength.—A magnetic field arises from the motion of electrical charges. This is fundamental to our present theories of magnetism. The magnetic field of a lodestone is due, we believe, to the integral effect of many electrical charges rotating in orbits within the atoms of the lodestone. A magnetic field has both direction and magnitude. This means that it can be treated vectorially. The *direction of a magnetic field* is that of a magnet suspended in the field when the positive end of the magnet is taken as the pointer end. The *magnitude* is called the *field strength, field intensity,* or *magnetizing force.* It is defined as the force in dynes exerted upon a unit magnetic pole placed at the point in the field where the intensity is being considered. H is a common designation for field strength. When H is constant over a large volume it is spoken of as a uniform field. From the definition of H and Coulomb's law it follows that

$$H = \frac{m}{d^2} \tag{3}$$

in vacuum or air. This is the defining equation for field strength, d being the distance from the pole m. At a distance of one centimeter from a unit magnetic pole the field intensity is unity, which, by Coulomb's law, means a force of one dyne per unit pole. Another way of expressing the unit of field intensity is to give it in terms of flux density. The maxwell (p. 8) has been

adopted as the unit of flux. If the flux density is one maxwell per square centimeter of cross-section it is called a flux density of one gauss, or field intensity of one gauss. When such is the case, the force exerted upon a unit magnetic pole in a field of this intensity is one dyne. It now becomes more evident than ever that the graphical representation of a magnetic field can be one of quantitative procedure. To represent a magnetic field draw as many lines per unit cross-section as are numerically equal to the field strength expressed in gausses. Figure 15 will thus represent a field strength of five gausses, and the direction of the field will be that of the lines drawn. Since the force in dynes exerted on a unit magnetic pole by a magnetic field of strength H is H, it follows that the force imposed on any magnetic pole of strength m is:

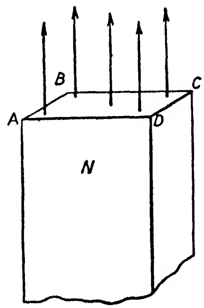


FIG. 15.—The area $ABCD$ represents 1 sq. cm., and each line a maxwell of flux. This figure, therefore, is a quantitative representation of a field intensity of five gausses.

$$F = Hm. \tag{4}$$

7. Gauss' Law.—The above discussion leads to the law of Gauss. This states that $4\pi m$ maxwells of flux emanate from a magnetic pole of strength m. If a spherical surface of one centimeter radius is drawn about a unit pole as a center there is at every point on the surface a field strength of one gauss. By the definition of field intensity the magnetic flux through each square centimeter of surface is one maxwell. Since the area of the unit sphere is 4π square centimeters, 4π maxwells is the total flux from the unit pole and from a pole of strength m, there will be a total flux of $4\pi m$ maxwells:

$$\Phi = 4\pi m \text{ maxwells.} \tag{5}$$

8. Magnetic Moment of a Magnet.—Let a bar magnet of pole strength m be suspended so as to swing freely in a horizontal plane in a uniform magnetic field of strength H (Fig. 16). Since the direction of H is that given by the arrows, the north-seeking pole will move in that direction and by the same token the south-seeking pole will move in the opposite direction. Inasmuch as the poles are inseparable, the magnet will come to rest with its axis parallel to the field. With the magnet in the position indicated in Fig. 16, there is acting upon the north pole a force

equal to $+Hm$, and on the south pole an equal but opposite force equal to $-Hm$. These two equal and parallel forces produce a couple whose turning moment is:

$$C = 2Hm(AO) = 2Hm(NO \sin \Theta). \tag{6}$$

This couple becomes equal to zero when $\Theta = 0$ and indicates why the magnet takes a position parallel to the field. When $\Theta = 90°$, $\sin \Theta = 1$, and

$$C = 2Hm(NO). \tag{7}$$

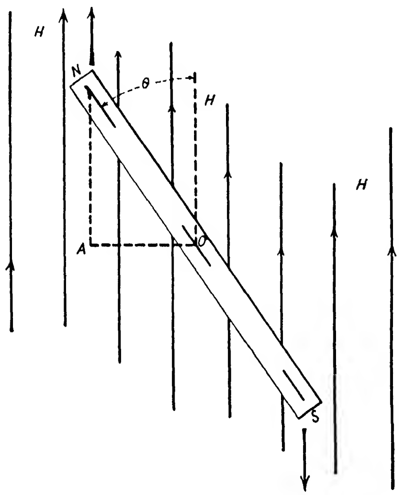


FIG. 16.—When a magnet is set at an angle θ with the impressed magnetic field, the action of the couple is to reduce θ to zero.

$2(NO)$ equals the distance between the poles of the magnet which is equal to NS,

$$\therefore C = Hm(NS). \tag{8}$$

If the field is such that $H = 1$, then

$$C = m(NS),$$

which has the dimensions of a moment of a force. This product of the pole strength of a magnet by the distance between its poles is called the *magnetic moment of a magnet* and is designated by the letter M. When $\Theta = 90°$,

$$C = HM. \tag{9}$$

This equation says that *the magnetic moment of a magnet is the moment of a couple, required to hold the magnet normal to a unit magnetic field.*

9. Period of a Magnet Freely Suspended in a Magnetic Field. In the equation, $C = MH \sin \Theta$, if Θ is small then $\sin \Theta$ may be expressed by Θ in radians, and

$$C = MH\Theta. \tag{10}$$

Since C varies as Θ, the equation shows that the freely suspended magnet will vibrate with SHM. The general equation for such motion is:

$$C = \frac{4\pi^2 K\Theta}{t^2} = MH\Theta,$$

or

$$t^2 = \frac{4\pi^2 K}{MH}$$

$$t = 2\pi\sqrt{\frac{K}{MH}}. \tag{11}$$

K = moment of inertia of the magnet.

10. Field Due to a Bar Magnet.—The greater part of the discussion thus far has treated the magnetic poles as though they were isolated. The only way in which this can be approximately realized experimentally is by taking very long, thin magnets. Since magnetic poles cannot be separated, it would be more in line with actuality to deal with the magnetic fields about magnets rather than with individual poles. Two points in the field of a magnet are of especial interest inasmuch as they are used very frequently in magnetic theory and practice. These are P and Q in Fig. 17. P is on a line bisecting the magnet, normal to its axis, while Q lies on an extension of the axis. By definition the field strength is the force on a unit magnetic pole so it will be understood that a positive pole is placed first at P and then at Q, to determine the force exerted upon it by the magnet NS. If h is half the length of the bar magnet and d the distance from P to the middle point of NS, then the force on the unit positive pole at P, due to the pole N of the magnet, will be m/PN^2, while the force due to the pole S will be m/PS^2. These two forces are represented by the two vectors PA and PB, while their resultant PH is the force F, acting on the unit magnetic pole at P, and therefore the field strength sought for. From the two triangles PHA and NSP there follows the relation:

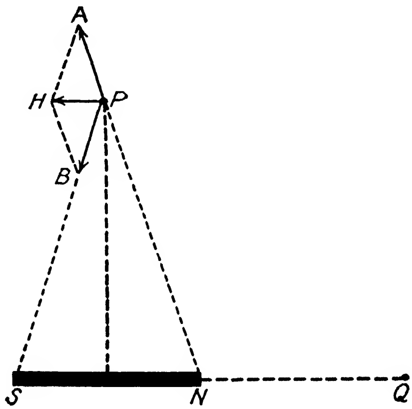


FIG. 17.—What forces are acting at the points P and Q due to a bar magnet, NS?

$$PH:NS = PA:PN,$$

but

$$PA = \frac{m}{PN^2} \text{ and } NS = 2h,$$

$$\therefore PH = \frac{2mh}{PN^3};$$

also since

$$PN^2 = d^2 + h^2, PN^3 = (d^2 + h^2)^{3/2},$$

PH must, therefore, equal $\dfrac{(2mh)}{(d^2 + h^2)^{3/2}}$,

or

$$PH = F_P = \frac{M}{(d^2 + h^2)^{3/2}}, \tag{12}$$

which is the strength of the field at P due to the bar magnet. When d^2 is large in comparison with h^2, the value of the field strength at P may be written:

$$F_P = \frac{M}{d^3}. \tag{13}$$

If the field at P is actually measured it must be remembered that the earth's magnetic field is omnipresent and that what one measures is F and the earth's field combined. It is possible in many cases to compensate for the earth's magnetic field.

For the point Q, on an extension of the axis of the magnet, the field may be found in the following way: the distances that N and S are away from the unit positive pole placed at Q, respectively are $(d - h)$ and $(d + h)$.

$$\text{Force at } Q \text{ due to } N = \frac{m}{(d - h)^2};$$

$$\text{Force at } Q \text{ due to } S = \frac{m}{(d + h)^2}.$$

These two vectors act in opposite directions, hence the resultant force

$$F_Q = \frac{m}{(d - h)^2} - \frac{m}{(d + h)^2} = \frac{m(d + h)^2 - m(d - h)^2}{(d^2 - h^2)^2};$$

$$F_Q = \frac{4mhd}{(d^2 - h^2)^2}. \tag{14}$$

This is the strength of the field at Q due to the magnet NS. Since $2mh = M$, the magnetic moment of the magnet, the strength of the field at Q may be written:

$$F_Q = \frac{2Md}{(d^2 - h^2)^2}. \tag{15}$$

If d is large in comparison with h then h^2 may be neglected and

$$F_Q = \frac{2M}{d^3}. \tag{16}$$

It is interesting to note the relation between Eqs. (13) and (16).

11. Oersted's Discovery: Magnetic Fields Due to Electric Currents.—In 1820 Oersted[1] discovered that an electric current

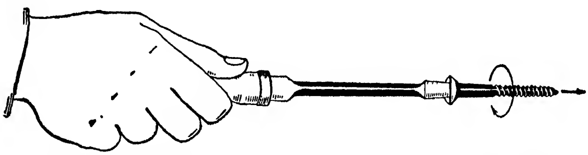


FIG. 18.—The progression of the screw corresponds to the direction of the electric current. The direction of rotation and the magnetic field are similarly related.

flowing in a conductor has a magnetic field surrounding it and that the force action is normal to it. Conventionally, the direction of the current and of the magnetic field are related to each other as the directions of translation and of rotation of a screw are associated (Fig. 18). If the conductor is bent into a

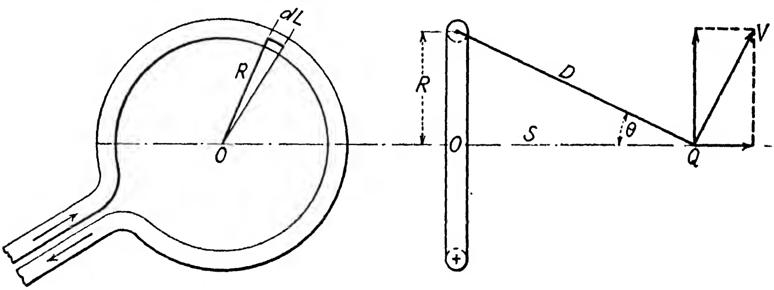


FIG. 19.—The magnetic field at the center of the coil is directed normally to the plane of the coil. In the coil to the left, the field is directed into the page. In the coil at the right, the field is directed to the right.

loop (Fig. 19), the magnetic field about the wire forms a field which links with the loop of wire in closed magnetic lines of force. The greater the number of loops of wire, the stronger will be the field at the center of the coil. This is one means whereby powerful magnetic fields may be developed. A coil of consider-

[1] OERSTED, *Gilbert's Ann.*, **66**, 295, 1820; "Experimenta circa Effectum Conflictus Electrici in Acum Magneticam," July 21, 1820; *Ann. Philos.*, **16**, 273, 1820.

able length, normal to the planes of the windings, is called a *solenoid*. Such a device will have its positive and negative magnetic poles and is the equivalent of a magnet. The introduction of an iron core increases the field inside the solenoid. This will be discussed later. If an iron ring has wire wound around the ring, as though a solenoid has been bent into a circle, the resulting device is called a *toroidal electromagnet* (Fig. 20).

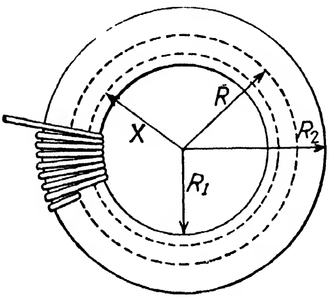


FIG. 20.—A toroidal electromagnet consists of an iron ring with wire wound about it as shown in the figure. No magnetic poles appear in a perfect toroidal electromagnet.

12. Ampère's Law.—Oersted showed that the magnetic-force action of a current flowing in a conductor is normal to the conductor. Since this magnetic field must react on other magnetic fields, Ampère[1] studied the effect of a magnetic field H on a conductor of length L carrying a current I when the conductor is normal to the field. He found that the force normal to the conductor could be expressed by the relation:

$$F = kIHL. \tag{17}$$

If the wire is not normal to the field, then the force acting upon the conductor is

$$F = kIHL \sin \phi. \tag{18}$$

By convention k in Eqs. (17) and (18) has been given a value of unity. This makes (17) a defining equation for the unit of electric current. The force in dynes with which one centimeter of the conductor is pushed sidewise by the unit magnetic field has been adopted as the fundamental measure of the strength of the current in the conductor. *A wire is said to carry a current of one absolute unit when one centimeter of the wire is pushed sidewise with a force of one dyne, when the wire is stretched across a magnetic field whose intensity is one gauss, the wire being normal to the field. The ampere is defined as one-tenth of this absolute unit of electric current.* The above definition of the absolute unit of current forms the basis of the so-called *electromagnetic system of electrical units.* It derives its name from the fact that the measurements rest upon the mechanical forces exerted by the magnetic fields of

[1] AMPÈRE, *Ann. chim. phys.*, **15**, 59, 1820.

the electric currents. This law of Ampère leads to the Biot-Savart law which is important for developing the equations giving the magnetic fields due to single conductors, coils, and solenoids.

13. Biot-Savart Law.—Biot and Savart[1] developed the law that at a perpendicular distance r from the center of a conductor, carrying a current I, the field strength due to an element of length dL of the conductor is:

$$\frac{kIdL}{r^2} = dH. \tag{19}$$

The unit of current may be so chosen that $k = 1$.

Equation (19) is a general equation whose validity cannot be tested directly by experiment, but if dH is integrated for some definite geometric form, the results may then be confirmed by laboratory methods. This will be done in succeeding paragraphs.

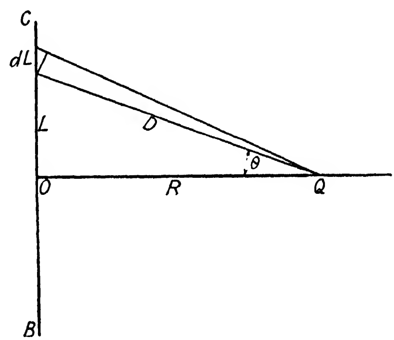


FIG. 21.—What is the magnetic field at Q due to an electric current flowing in an infinitely long conductor BC?

14. Magnetic Field Strength in the Neighborhood of an Infinitely Long Conductor Carrying a Current.—An infinitely long conductor CB (Fig. 21) has an electric current flowing in it. D is the distance from the point Q, where the field strength H is to be determined, to the element dL, and R is the perpendicular distance from the wire to the point Q. The effective length of dL is $dL \cos \Theta$, whence,

$$dH_Q = \frac{IdL}{D^2}\cos\Theta. \tag{20}$$

$$H_Q = \int_0^\infty \frac{IdL}{D^2}\cos\Theta = 2IR\int_0^\infty \frac{dL}{(R^2 + L^2)^{3/2}}.$$

Result is doubled because half the wire lies below zero.

$$H_Q = 2IR\left[\frac{L}{R^2(R^2 + L^2)^{1/2}}\right]_0^\infty = 2IR\left[\frac{1}{R^2\left(\frac{R^2}{L^2} + 1\right)^{1/2}}\right]_{L=\infty}$$

$$= \frac{2IR}{R^2} = \frac{2I}{R}.\text{[2]} \tag{21}$$

[1] BIOT and SAVART, *Ann. chim. phys.*, **18**, 222, 1820; BIOT, "Precis Elementaire de Physique," vol. II, p. 123.

[2] HUMPHREYS, *Science*, **26**, 417, 1907.

Maxwell[1] has given a very beautiful experimental proof of the inverse law for R in this last equation. Northrup[2] has shown that the magnetic field inside a conductor follows the law,

$$H_a = \frac{2Ia}{R^2}, \tag{22}$$

where a equals the distance from the center of the conductor, and R is the diameter of the conductor. Just at the surface of the conductor, (22) becomes equal to (21) (see Fig. 44).

15. Magnetic Field Strength Along the Axis of a Circular Current.—If dH in Eq. (20) is integrated along a conductor bent into a circular loop, as in Fig. 19 or 22, the values of the field strength along the axis of the loop may be obtained.

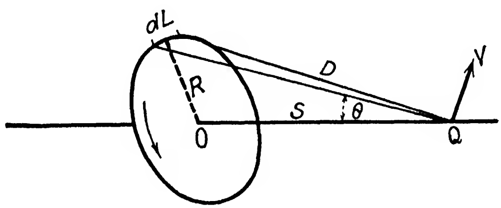


FIG. 22.—What is the magnetic field at the point Q due to an electric current flowing in a circular conductor, an element of which is dL?

The force at Q due to an element of current dL is given by the equation:

$$dH = \frac{IdL}{D^2}. \tag{23}$$

With the current flowing in the loop as indicated by the arrow in Fig. 19 the force action at Q due to the element dL is in the direction QV. The component of this force along the axis is $QV \sin \Theta$ or in terms of Eq. (23):

$$dH_Q = \frac{IdL}{D^2}\frac{R}{D} = \frac{IRdL}{D^3} = \frac{IRdL}{(R^2 + S^2)^{3/2}}. \tag{24}$$

Integrating this value around the loop,

$$H_Q = \frac{IR}{(R^2+S^2)^{3/2}}\int_{l=0}^{l=2\pi R} DL = \frac{IR}{(R^2+S^2)^{3/2}}(2\pi R) = \frac{2\pi IR^2}{(R^2+S^2)^{3/2}}. \tag{25}$$

This is general for any point along the axis. As a special case make $S = 0$ and Q will be at the center of the loop.

$$H = \frac{2\pi I}{R}. \tag{26}$$

[1] MAXWELL, "Electricity and Magnetism," 2d ed., vol. II, p. 130.

[2] NORTHRUP, *Phys. Rev.*, **24**, 474, 1907.

For n turns in the coil,

$$H = \frac{2\pi nI}{R} = \frac{2\pi nI}{10R} \text{ gausses,} \tag{27}$$

when I is expressed in amperes.[1]

Equation (27) may be used for calculating the field at the center of a tangent galvanometer[2] coil or modified for the Helmholtz[3] double coil.

In considering the force at the center of a coil of wire, like a tangent galvanometer coil, it may be of help to treat the prob-

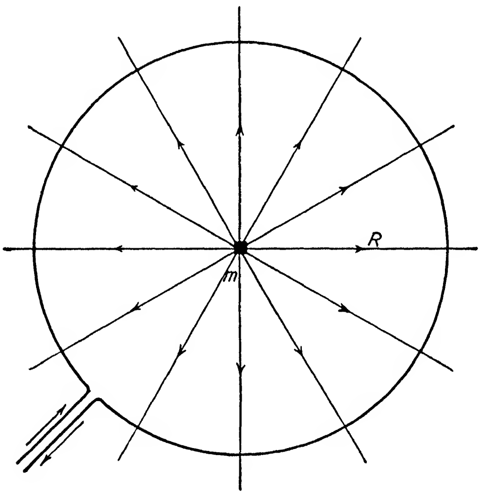


FIG. 23.—Action and reaction are equal. The magnetic pole m at the center of the loop exerts a force on the magnetic field set up by the current flowing in the loop.

lem in another way. The field at the center of the coil is the force in dynes acting on a unit magnetic pole at the center. Since action and reaction are equal, the force exerted by the coil on the magnetic pole must be the same as the force which the magnetic pole exerts on the coil. Let m (Fig. 23) be a magnetic pole placed at the center of the coil. At a distance R from m, *i.e.*, the radius of the coil, the magnetic field due to m is m/R^2.

[1] STROM, *Jour. Franklin Inst.*, **206**, 339, 1928.

[2] SMITH, "Electrical and Magnetic Measurements," p. 145, 1917; HADLEY, "Magnetism and Electricity for Students," p. 274, 1913.

[3] GAUGAIN and HELMHOLTZ, *Compt. rend.*, **36**, 191, 1853; *Poggendorff Ann.* **88**, 442, 1853.

By *Ampere's law,*

$$F = HLI,$$

$$\therefore F = \frac{m}{R^2}(2\pi nR)I \tag{28}$$

$$= \frac{2\pi mnI,}{R}$$

or for a unit pole,

$$H_Q = \frac{2\pi nI}{R} = \frac{2\pi nI}{10R} \text{ gausses.} \tag{29}$$

Compare Eq. (29) with Eq. (27).

16. Magnetizing Force at Any Point on the Axis of a Solenoid. A long spiral of wire is a succession of galvanometer coils placed

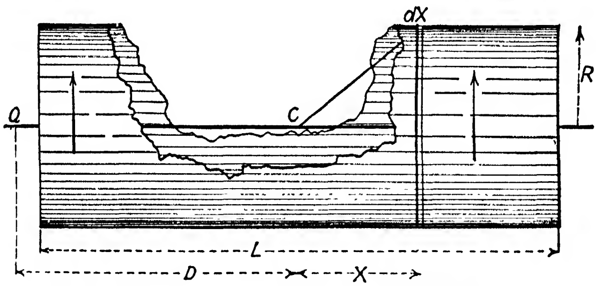


FIG. 24.—A solenoid. The field at the center, $H_c = \frac{4\pi nI}{10}$ gausses, when R is small with respect to L.

side by side along a common axis. Such a series of coils is called a solenoid. The Biot-Savart law is applicable in evaluating the field strength at points along the axis. Let R (Fig. 24) be the radius of a single layer of n turns per unit length, and L the length of the coil. What will be the magnetic force on a unit magnetic pole at a point Q, whose distance from the center of the coil C is D? In the element of length of coil, dX, there are ndX turns. The effect of these turns at the point Q is shown by the equation from which (25) was derived, to be;

$$dH_Q = \frac{2\pi IR^2ndX}{[(D + X)^2 + R^2]^{3/2}}. \tag{30}$$

If the effect at Q of all of the elements dX along the coil are summed up, there results:

$$H_Q = 2\pi nIR^2 \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dX}{[(D + X)^2 + R^2]^{3/2}} \tag{31}$$

$$= 2\pi nIR^2 \left[\frac{D + X}{R^2\sqrt{(D + X)^2 + R^2}} \right]_{-\frac{L}{2}}^{+\frac{L}{2}}$$

$$= 2\pi nI \left[\frac{D + \frac{L}{2}}{\sqrt{\left(D + \frac{L}{2}\right)^2 + R^2}} - \frac{D - \frac{L}{2}}{\sqrt{\left(D - \frac{L}{2}\right)^2 + R^2}} \right]. \tag{32}$$

If $D = L/2$, Q is at one or the other ends of the solenoid, and

$$H_Q = 2\pi nI \frac{L}{\sqrt{L^2 + R^2}}. \tag{33}$$

If $D = 0$, Q is at the center, and

$$H_Q = 4\pi nI \frac{L}{\sqrt{L^2 + R^2}}. \tag{34}$$

From these two equations it is evident that H in (33) is just half the value of H in (34), the values of the magnetizing force at the ends and the center of the coil respectively.[1] For long, slim coils, *i.e.*, L very large with respect to R, the field at the center is quite accurately represented by the equation:[2]

$$\begin{aligned} H &= 4\pi nI \\ &= 0.4\pi nI \text{ gausses,} \end{aligned} \tag{35}$$

when I is expressed in amperes.

In calculating the constant of a solenoid from its dimensions, the values of H should be worked out for each layer if the coil is made up of several layers.

Consideration of Eq. (35) indicates that there are two ways in which the field along the axis may be increased, either by augmenting the number of turns or by developing more current in the coil. Increasing the number of turns increases the resistance of the coil. Limitations to this procedure are very quickly reached. On the other hand, increasing the current causes the heating effects to rise very rapidly. Also, the mechanical forces exerted on the turns of the coil become terrific for large currents. By means of iron cores, fields of 50,000 gausses have been maintained over very small volumes between the poles of an electro-

[1] Williams, *Jour. Franklin Inst.*, **182**, 359, 1916.

[2] Starling, "Electricity and Magnetism," pp. 228–229, 1924.

magnet. To obtain *permanent* fields of several hundred thousand gausses, say 300,000 to 500,000, involves technical difficulties which up to the present have been insurmountable. Thus far *intense magnetic fields* lasting for only a fraction of a second have been obtained. The problem of producing intense magnetic fields, constant for any length of time, forms one of the very important problems in magnetism. Could one produce an alloy or material whose permeability was only five to six times that of the best iron, a great step in the right direction would be made.

The Mount Wilson Solar Observatory has developed a coil for producing magnetic fields of over 30,000 gausses without iron cores. The solenoid is wound in layers of bare copper tape, separated by small cord. Through the interspaces of tape and cord, kerosene is rapidly forced from one end of the coil to the other by means of a pressure pump. After passing through the coil the kerosene is conducted through a refrigerator. The main coil is built with an inner and outer section which may be connected in parallel. Each section carries 2,000 amperes, so that the whole takes 4,000 amperes at 125 volts from a specially constructed generator. Thus excited, the 5-cm. tubular space inside has a field of about 32,000 gausses over about 10 cm. of its length. This is not an intense magnetic field, but it is fairly uniform over a comparatively large volume and can be maintained indefinitely.

Very large transient fields have been obtained by the surge of enormous electric currents through specially constructed coils. The source of power in some cases has been large capacities; in others, storage batteries of large capacities. The most successful, however, has been used by Kapitza,[1] working in the Cavendish Laboratory. His source of power lies in the accumulation of kinetic energy in the rotor of a specially devised turbo-generator. When the generator is short-circuited through the magnetizing coil the kinetic energy of the rotor is converted into electrical power. In order to avoid undue heating effects the energy of only half a cycle was used. This had the added advantage that the circuit could be broken at the end of the half cycle without the difficulties which were encountered in breaking the huge currents when a storage battery was the source of energy. Kapitza figured that a generator giving 2,000 kw. on continuous service would be feasible. This meant that special

[1] Kapitza, *Proc. Roy. Soc.*, **115**, 658, 1927;
Wall, *Jour. Inst. Elec. Eng.*, **64**, 745, 1926;
Cockcroft, *Philos. Trans.*, *A*654, **227**, 317–343, 1928.

attention had to be paid to its construction from a mechanical point of view. The windings were so made that instead of a sinusoidal form of alternating-current, the crest of the wave would be flat. During this portion of the cycle the current was sent through the coil and thus a constant field was obtained during that short interval of time. For many experiments, like bending the tracks of the α particles or the Zeemann effect, this short interval of time was sufficient. A very powerful and quick-acting switch was built to throw in and out the desirable and undesirable portions of the cycle in order to get the level portion of the cycle. Furthermore, the current in the magnetizing coil had to be in step with the observing device. This necessitated an accurate timing arrangement between the switch and the observing outfit. With the huge currents sent through the magnetizing coil, special precautions had to be taken against the bursting of the coil due to the terrific electrodynamic forces present. One is at a loss to know whether to admire more the final observations[1] taken with the completed outfit, or the patience, labor, and infinite care used in developing the outfit. Both are excellent problems of research. Transient fields of 320 kilogausses were obtained and calculations indicated that 500 kilogausses were possible. Progress in the development of intense magnetic fields has very great significance in the development of our atomic theories.

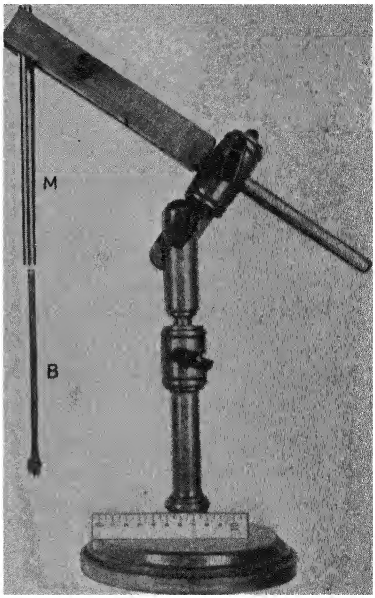


FIG. 25.—The soft iron rod B is separated from the magnet M by a sheet of paper, yet the filings adhere to the lower end of B.

17. Magnetization by Induction.—In Sec. 1 the fact was mentioned that magnetization may be produced by rubbing a steel rod with a piece of magnetite. It may also be produced by the *process of induction.* In Fig. 25 a piece of soft iron B, when dipped into iron filings, will not attract the filings. If a bar

[1] KAPITZA, *Proc. Roy. Soc.*, **106**, 602, 1925.

magnet M is held end-on to the upper end of the soft iron rod immediately the soft iron takes on the property of attracting the iron filings. By the definition of magnetization this means the soft iron has been induced into a state of magnetization by the presence of a magnetic field. When the bar magnet is removed the soft iron specimen will lose wholly or in part the magnetization imparted to it by induction. In Sec. **11** it was pointed out that a coil of wire is the equivalent of a magnet. A solenoid or any other form of a coil may be brought into the neighborhood of the soft iron and, when an electric current is flowing through the coil, magnetization will be induced in the soft iron. Since for substances like iron, nickel, and cobalt, the permeability is greater than unity, the magnetization of a piece of iron by induction means that the field inside of a coil may be greatly enhanced by a core of iron.

The same units are applied to this magnetic flux in iron as in air, *viz.*, the unit of induced magnetic flux is called the maxwell and the unit of induction flux density the gauss. If B is the induction density in the iron after being exposed to a magnetizing force H, then

$$\frac{B}{H} = \mu, \tag{36}$$

which is the same permeability factor which appeared in Eq. (2). If A is the cross-section of the iron core in the solenoid the total flux through the iron will be:

$$\begin{aligned} \phi_{\text{iron}} &= BA \\ &= \mu HA. \end{aligned} \tag{37}$$

For the tubular space within a long solenoid filled with air,

$$\phi_{\text{air}} = HA; \qquad \frac{\phi_{\text{iron}}}{\phi_{\text{air}}} = \mu, \tag{38}$$

where A is the cross-section of the space inside the solenoid.

18. Electromagnets.—A solenoid with an iron core is called an electromagnet. A very simple form is illustrated by Fig. 26. If the iron core, with the solenoid surrounding it, is bent into a circle so that the iron forms a closed ring, there results what was called in Sec. **11** a toroidal electromagnet. It has many important commercial uses. So far as the path of the magnetic lines of force is concerned the toroidal electromagnet is the best type of all forms.[1] If a perfect toroidal electromagnet could be

[1] UNDERHILL, "Magnets," p. 38, 1924.

built it would exhibit no poles and, therefore, no stray magnetic fields about it. Wherever stray fields occur, there *leakage* of the

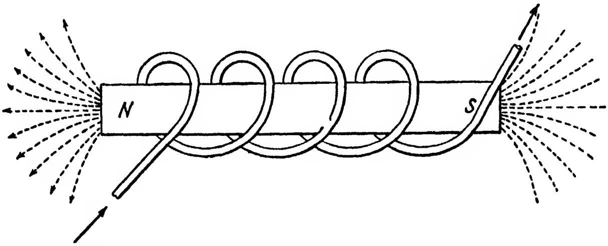


FIG. 26.—An electromagnet. A solenoid with an iron core constitutes an electromagnet.

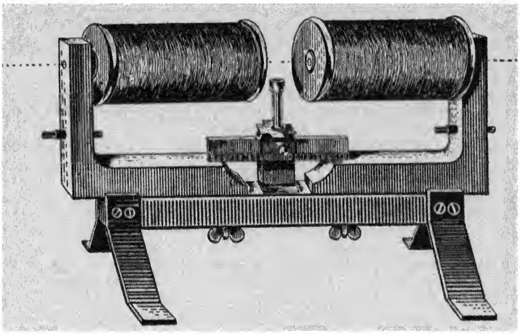


FIG. 27.—A useful form of electromagnet. Hollow cores for optical experiments.

lines of force is said to exist. In an efficient electromagnet this must be reduced to a minimum. The best construction of an electromagnet for experimental purposes will attempt, therefore, to follow the lines of a toroidal electromagnet as far as possible. For experimental purposes there must be somewhere in the magnetic circuit a place where the substance being investigated may be exposed to the influence of the magnetic field. This is done by removing a small section of the toroidal iron core and making the corresponding air gap a part of the magnetic circuit. The ends of the iron core form the magnetic poles of the circuit whereby

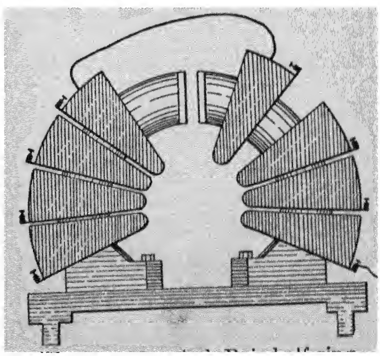


FIG. 28.—A duBois half-ring electromagnet.

powerful magnetic fields are developed. If the air gap is made very narrow, practically the same magnetic flux intensity will exist there as occurs in the iron core. Figures 29, 30, and 31 show various types of experimental electromagnets. Figure 27 shows an electromagnet suggested by Ruhmkorff[1] which is very

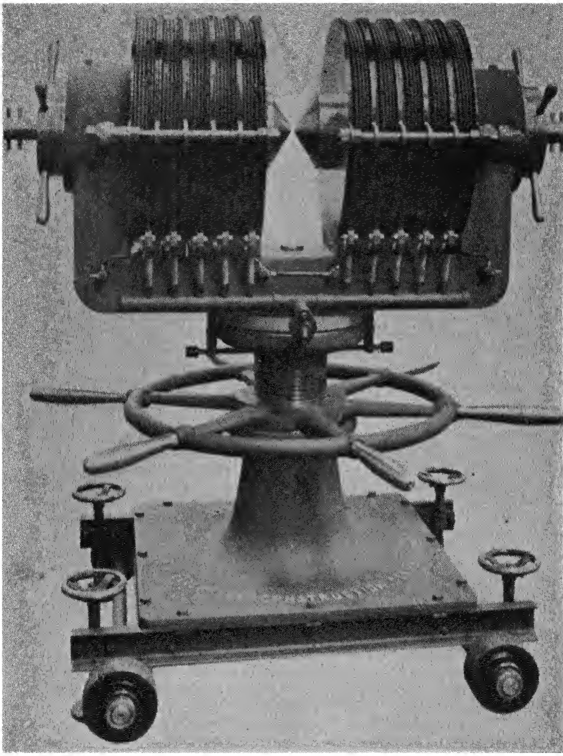


FIG. 29.—An electromagnet whose coils are wound with copper tubing for cooling with water.

useful for magneto-optical experiments in which the light passes through the specimen in a direction parallel to that of the magnetic field. This is accomplished by boring holes through the pole pieces in the same direction as the field. Figure 28 illustrates the half-ring electromagnet developed by duBois.[2] It is a powerful electromagnet. In trying to develop strong magnetic

[1] RUHMKORFF, *Compt. rend.*, **23**, 417, 538, 1846.
[2] DUBOIS, *Wiedemann Ann.*, **51**, 537, 1894.

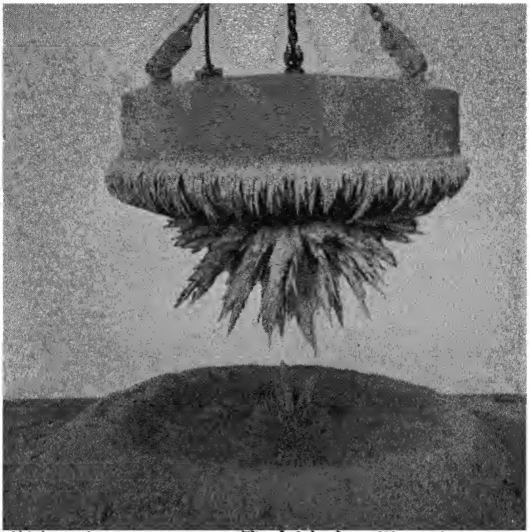


FIG. 30.—A lifting electromagnet. Useful in handling pig and scrap iron.

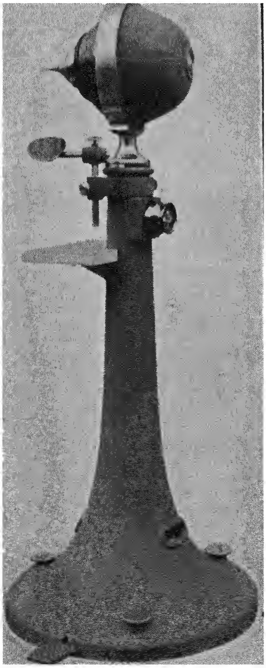


FIG. 31.—An electromagnet used by oculists for extracting iron particles from the eye.

fields Weiss[1] paid particular attention to the material of the pole-pieces. Ferrocobalt gave the best results. Weiss employed large electric currents for exciting his electromagnet. In order to keep the temperature down as much as possible, the coils were wound with hollow copper wire through which cold water could be circulated. Fifty to 60 kilogausses were thus obtained. The problem of maintaining intense magnetic fields[2] over long periods

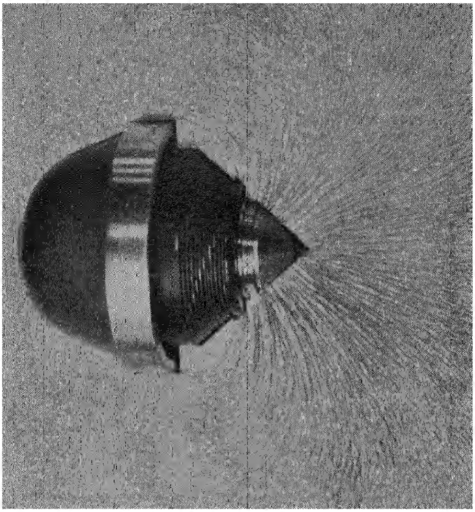


FIG. 32.—Showing the intense magnetic field developed at the pole of the electromagnet illustrated in Fig. 31.

is still a live one (see Fig. 29). Some very unusual forms of electromagnets will be discovered in a book on electromagnets by S. P. Thompson. Figures 30, 31, and 32 show some special applications of electromagnets.

19. Intensity of Magnetization and Magnetic Susceptibility.—Figure 12 showed that the magnetic lines of force seemed to draw in and pass through a para- or ferromagnetic body with greater density of distribution than through the same space filled with air. Figure 35 shows the resultant field about an iron bar when it is introduced into a uniform magnetic field. The lines of

[1] WEISS, *Jour. de phys.*, **6**, 353, 1907.

[2] HOUSTON and KENNELLY, "Magnetism," p. 183, 1906;
DESLANDRES and PERROT, *Compt. rend.*, **158**, 226, 1914.

force pass through the iron more readily than the air. Let the bar in Fig. 26 be simply one thrust into a solenoid whose field is uniform. The iron being magnetized by induction will have positive

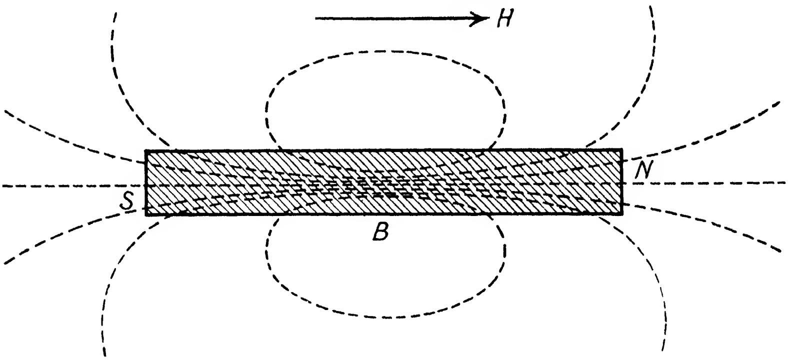


FIG. 33.—H represents the direction of the imposed field. N and S are the magnetic poles induced in the rod B. Since magnetic lines of force are closed lines, their return paths are in a direction opposite to H, hence a demagnetizing effect.

and negative magnetic poles, the same as a magnet produced by stroking. From and to these same poles will emerge and enter

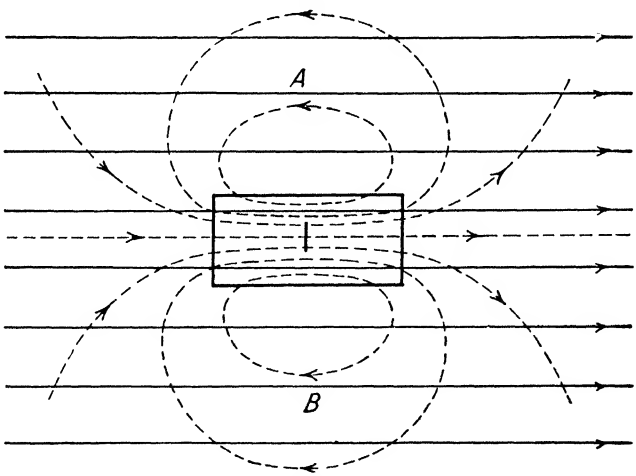


FIG. 34.—The induced and inducing magnetic fields oppose each other at points A and B. The dotted lines indicate the induced field, which is sometimes called the polar field.

lines of force just as they do from any magnet. Figure 33 shows these lines of force in a uniform field of strength H. The lines of force due to H are not shown. If the lines of force from the

solenoid and those from the induced magnet are superimposed on each other as in Fig. 34 it will be evident that at A and B the two fields are opposing each other. Not only is this true in the free space between the solenoid and the iron but it is also true within the bar itself. In other words the induced magnetization not only tends to cut the effectiveness of the magnetizing force but actually demagnetizes, to a certain extent, the induced magnetism of the rod. This *demagnetizing effect* is seen in Fig. 35

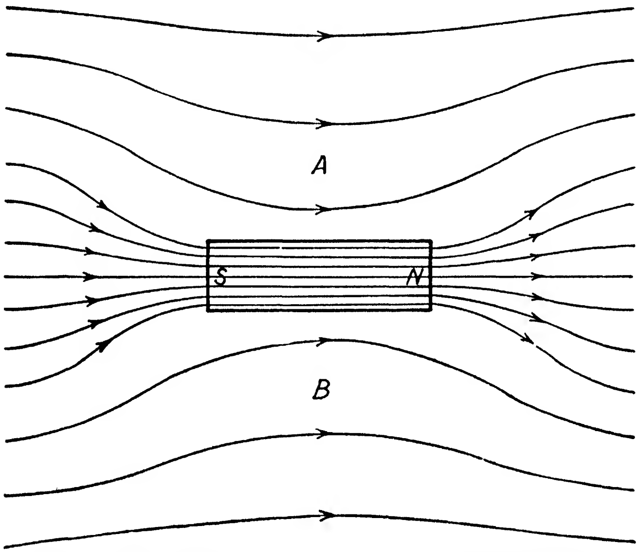


FIG. 35.—The resultant of the induced and imposed fields.

where the lines of force are farther apart at A and B than elsewhere in the field.

Directly at the poles where both sets of lines coincide, the excess number of lines over those which would be there if the iron were not present may be expressed in terms of the pole strengths which have been induced. This may be done in terms of Gauss' law. If the pole strengths of the induced magnet are m, then the relation holds that

$$\phi_{\text{iron}} - \phi_{\text{air}} = 4\pi m \tag{39}$$

(assuming that all the lines of force emanate from the ends of the poles). Equation (39) is expressed in terms of the total flux. In terms of the flux density we have:

$$\frac{\phi_{\text{iron}}}{A} - \frac{\phi_{\text{air}}}{A} = \frac{4\pi m}{A},$$

or

$$B - H = \frac{4\pi m}{A}, \tag{40}$$

where A is the cross-section of the iron core. To the quantity m/A has been given the special name, *Intensity of Magnetization.* This is frequently designated by the script letter $\mathcal{I}$, so that

$$B - H = 4\pi\mathcal{I}$$

or

$$B = H + 4\pi\mathcal{I}. \tag{41}$$

This equation states that the lines of flux through a bar of iron placed in a uniform magnetic field is composed of two parts, the magnetic lines of force due to the imposed field and the resultant lines of magnetization produced by the orientation of electronic circuits within the iron. m/A is the pole strength per unit area of end of iron rod.
Since

$$\frac{m}{A} = \mathcal{I}, \tag{42}$$

the intensity of magnetization may be defined as the pole strength divided by the cross-section.

Taking L as the length of the induced magnet and using the relation given in (42) one may derive the following relationship:

$$\mathcal{I} = \frac{mL}{AL} = \frac{M}{V}. \tag{43}$$

This is another definition for $\mathcal{I}$ and states that *the intensity of magnetization is the magnetic moment per unit volume of the iron rod.*

Instead of referring the intensity of magnetization to unit volume it is a more modern practice to refer $\mathcal{I}$ to unit mass, whence

$$J = \frac{M}{V\rho} = \frac{\mathcal{I}}{\rho}. \tag{44}$$

J is called the *specific intensity of magnetization,* ρ being the density of the material. When J is multiplied by the molecular or atomic weight of a substance the products are the so-called *molecular and atomic intensities of magnetization respectively.*

Dividing Eq. (41) by H gives the form,

$$\frac{B}{H} = \frac{H}{H} + \frac{4\pi\mathcal{I}}{H},$$

or

$$\mu = 1 + 4\pi K. \tag{45}$$

Just as $B/H = \mu$ has a special significance, so $\mathcal{I}/H = K$ does also, and is called the *magnetic susceptibility* of a substance. The susceptibility may also be referred to unit mass and called the *specific susceptibility* or *susceptibility per unit mass*. In which case,

$$\chi = \frac{J}{H} = \frac{\mathcal{I}}{H\rho} = \frac{K}{\rho}. \tag{46}$$

χ multiplied by either the molecular or the atomic weight gives the *molecular and atomic susceptibilities* respectively. In comparing different substances it is customary to use the gram-atomic or gram-molecular weights as standards of mass. Consequently, if we wish to express the volume susceptibility in terms of atomic susceptibility, we multiply volume susceptibility at 0° C. and 76 cm. pressure by 22.4×10^3. μ measures how permeable a substance is to the magnetic lines of force, while K indicates how susceptible the substance is to the influence of a magnetizing field.

The value of K is, as it were, a measure of the magnetizing effect of a magnetic field on the material placed in this field. If, therefore, we are comparing the magnetic properties of different substances we will be much more interested in the susceptibility than in the permeability.[1]

In speaking of lines of force, magnetization, and induction, the following designations will help to keep the distinctions clear,

1. *Lines of force* apply to the magnetic field produced by some outside agent. The field of a solenoid is an illustration.

2. *Lines of magnetization* are the resultant lines due to the orientation of electronic circuits within a substance when it is magnetized. They are represented by the term $4\pi\mathcal{I}$ in Eq. (41).

3. *Lines of induction* or magnetic flux is the term applied to the whole group made up of 1 and 2. They are denoted by the symbol B in Eq. (41).

Equation (41), therefore, may be expressed in words as follows:

$$\left.\begin{array}{l}\text{Lines of induction}\\ \text{(or magnetic flux)}\end{array}\right\} = \text{lines of force} + \text{lines of magnetization.}$$

In quality they are all the same because the lines of force are due to electrons moving in the magnetizing coil and the lines of magnetization are due to electrons moving in atomic orbits.

[1] Dushman, "Theories of Magnetism," *Gen. Elec. Rev.*, May, August, September, October, and December, 1916.

As an aid in picturing the magnetic flux through a piece of iron, the following point of view[1] may be observed. A block of iron is placed in a uniform magnetic field of intensity H parallel to the direction of the arrow (Fig. 36). From this block of iron a very long and slim cylindrical volume has been removed. The

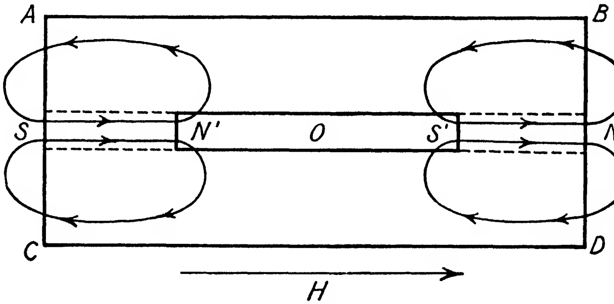


FIG. 36.—A block of iron $ABCD$ has a slim cylindrical portion $N'OS'$ removed from it. When placed in a uniform field of strength H, the field at O will also be H.

axis of the cylinder is parallel to H. By induction, magnetic poles will be induced at the ends of the cylinders of iron of which the hollow space is an extension. These will be S, N', S', and N, respectively. These magnetic poles radiate lines of magnetiza-

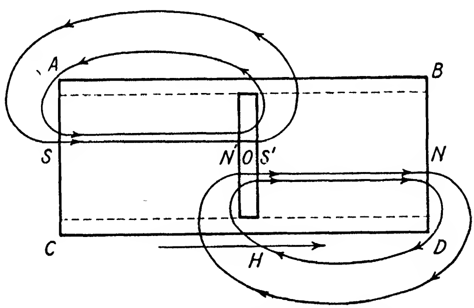


FIG. 36a.—A block of iron $ABCD$ has a narrow crevasse $N'OS'$ removed from it. When placed in a uniform field of strength H, the field at O will be $H + 4\pi\mathcal{I}$.

tion as shown by the curved lines in Fig. 36. Some of these reach out into the empty cylindrical space, but if the cylinder is long enough, these will not go as far as the center of the free space. Consequently the field at O will be that due to the original field H. At the point O the equation, $B = H + 4\pi\mathcal{I}$ takes the form $B = H$ since $\mathcal{I}$ at that point equals zero. On the other

[1] MAXWELL, "Electricity and Magnetism," 3d ed., vol. II, p. 24.

hand, if a section of the iron, the size of a thin silver coin, is removed as in Fig. 36*a*, the result at the center of this free space is quite different. Once more magnetic poles S, N', S', and N will be formed at the ends of the cylinders of iron which have the diameter of the free space. The lines of magnetization which radiate out from these poles will reach across the free space. Not only are the lines of force H passing through the narrow crevasse, but also the lines of magnetization from the induced poles go through in the same direction. If we can get the number of lines of magnetization passing through the free space we can give the total flux through that same space. This can be done by invoking the aid of Gauss' law. The two pole surfaces N' and S' may be thought of as plane sheets of positive and negative poles, wherein $\mathcal{J}$ equals the pole strength per unit area of sheet.

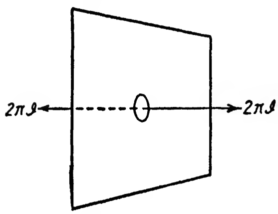


FIG. 36*b*.—Radiation of $4\pi\mathcal{J}$ lines of magnetization from unit area.

Ampère[1] has stated that "every linear conductor carrying a current is equivalent to a simple *magnetic shell*, the bounding edge of which coincides with the conductor, and the moment of which per unit area, *i.e.*, the strength of the shell, is proportional to the strength of the current." By a *magnetic shell* is meant an infinitely thin sheet of material, magnetized in a direction normal to the plane of the sheet, so that one side is a N, and the other a S polar surface or sheet.

According to Gauss' law, $4\pi m$ lines of force emerge from a magnetic pole of strength m. In the case of a magnetic polar sheet (Fig. 36*b*), as many lines of force radiate normally from one side of the sheet as from the other. Think of a cylinder cutting the sheet normal to its plane. Let the cross-section of the cylinder be A. Since $\mathcal{J}$ is the pole strength per unit area it will be the surface density of the magnetic polar sheet. In the part of the sheet cut out by the cylinder there will be a quantity of $A\mathcal{J}$ poles. According to Gauss' law the lines of force which go out from such a polar sheet will be:

$$2HA = 4\pi\mathcal{J}A,$$

or

$$H = 2\pi\mathcal{J},$$

[1] AMPÈRE, "Théorie des phénomènes électro-dynamique," *Mém. de l'insti.*, No. 4, 1823.

the magnetic force on each side of the sheet. Since $2\pi\mathcal{I}$ lines of force pass out from each unit area of each side of the polar sheet, the value of one side will be $2\pi\mathcal{I}$ and the other $-2\pi\mathcal{I}$, making a total of $4\pi\mathcal{I}$ in conformity with Gauss' law. In the bounding faces of the free space in Fig. 36*a*, there are two magnetic polar sheets of opposite signs. Being of opposite signs, the lines of magnetization will be in the same direction and equal to $4\pi\mathcal{I}$. Therefore, the total flux is

$$B = H + 4\pi\mathcal{I}.$$

In a free space, such as that in Fig. 36*a*, the flux across the narrow crevasse is the same as through the main body of the iron. At the center of the long cylinder in Fig. 36, the flux is the same as though the iron were not present. This indicates why the poles of an electromagnet should be brought very close together, *i.e.*, with a narrow crevasse between them, in order to get the most intense field.

20. Demagnetizing Factors.—In the preceding section it was shown that the introduction of a piece of iron into a solenoid decreased the effective magnetizing power of the coil. This arose from the fact that the poles, induced in the iron, set up lines of magnetization which were opposed to the magnetizing force of the solenoid. A better way of expressing this is to say that, due to the orientation of electronic circuits in the iron, the return lines of magnetization are set to oppose the magnetizing force of the solenoid. When one is studying the magnetic intensity of a ferromagnetic body this demagnetizing effect must be taken into account because the field of the coil, $H = 4\pi n\mathcal{I}$, is not the effective magnetizing force. This demagnetizing effect, due to the induced magnetic poles, can be computed fairly accurately. It will depend largely upon the form of the specimen used. If the material used is in the form of an ellipsoid of rotation, the demagnetization will be a function of the ratio,

$$r = \frac{\text{length}}{\text{diameter}} = \frac{L}{D}.$$

This means that for long, slim wires the demagnetization effect will be very small. The counter field set up by the induced poles will be a function of the pole strengths. These are proportional to $\mathcal{I}$, the intensity of magnetization of the iron. The effective magnetizing force may, therefore, be expressed as follows:

$$H = H' - N\mathcal{I}, \tag{47}$$

where H' is the field of the solenoid without the iron and N is the factor whereby $\mathcal{I}$ must be multiplied in order to get the amount of demagnetization. N is the so-called *demagnetizing factor*.

Maxwell[1] shows that N may be calculated by the formula,

$$N = 4\pi\left(\frac{1}{E^2} - 1\right)\left(\frac{1}{2E}\log\frac{1+E}{1-E} - 1\right), \tag{48}$$

where the specimen is an ellipsoid having semi-axes a, b, and c, whose relations are:

$$a = b = c\sqrt{1 - e^2}.$$

The advantage of dealing with an ellipsoid lies in the uniform magnetic induction passing through it when placed in a uniform field. A long, slim wire approximates an ellipsoidal form. If to an iron wire there are given the values, $r = 500$ and $K = 200$, then

$$\begin{aligned} H &= H' - 0.00030\mathcal{I} \\ \frac{H'}{H} &= 1 + 0.00030K \\ \frac{H'}{H} &= 1.06. \end{aligned} \tag{49}$$

Even with r as large as 500 the effective field is 6 per cent less than the applied field and indicates how necessary it is to correct for the demagnetizing effect of the polar field. In the case of thick, short specimens this demagnetizing effect is so great that the effective magnetizing force is a very small part of that produced by the solenoid.

A thin sheet of magnetic material, normal to the magnetic field, has the largest demagnetizing field of all. $N = 4\pi$. For a permeability of 1,000 the effective field inside the sheet is only 1/1,000 of that outside.

Applying the demagnetizing correction to the relations between B and H amounts to taking the apparent curve of magnetization and *shearing it back*, so that it becomes a normal curve of magnetization. Lord Rayleigh[2] first called attention to the fact that from a knowledge of N a hysteresis curve, obtained from an ellipsoid of iron and plotted as a BH curve, could be sheared

[1] MAXWELL, "Electricity and Magnetism," 2d ed., vol. II, p. 65.

[2] RAYLEIGH, *Philos. Mag.*, **22**, 175, 1886;

SCHMIDT, "Magnetische Untersuchung," "Encyklop. der Elektrochemie," vol. XI, 1900.

back into normal hysteresis curve. The normal curve would be for an ellipsoid of the same cross-section but infinitely long. Figure 37 indicates the manner in which the various apparent induction curves approach the normal one as r grows larger and larger. A similar diagram could be shown for hysteresis curves as well. Practically, a normal curve could be obtained with the material in the form of a toroidal electromagnet since no poles are formed. By experimental methods various investigators[1] have worked out demagnetizing factors for various kinds and forms of ferromagnetic substances. At first it was thought that N was constant and depended only on the ratio of L/D. Investigations

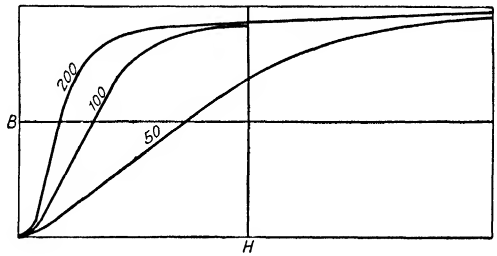


FIG. 37.—As the ratio of length to diameter increases, the induction curve approaches more and more to that of the normal curve, *i.e.*, there is less demagnetization.

by Benedicks[2] and Shuddemagen[3] have shown, however, that N is only approximately constant in the interval from $\mathfrak{I} = 100$ to $\mathfrak{I} = 800$. "Beyond this it decreases rapidly to a minimum value as the magnetization approaches saturation. Moreover, the N varies also for different absolute values of D, the diameter of the rod, being somewhat less for thick rods than for thin ones when L/D remains constant."

Instead of writing:

$$H = H' - N\mathfrak{I},$$

it may be written:

$$H = H' - CB, \tag{50}$$

depending upon whether one is working with $\mathfrak{I}$ or B. A set of values for N and C determined by Shuddemagen is given in the appendix of this book. Among other investigators of this subject

[1] DUSSLER, *Ann. der Phys.*, **86**, 66, 1928.
[2] BENEDICKS, *Wiedemann Ann.*, **6**, 726, 1901.
[3] SHUDDEMAGEN, *Phys. Rev.*, **31**, 165, 1910.

may be mentioned duBois,[1] Ewing,[2] Tanakadate,[3] Mann,[4] and Wurschmidt.[5] The latter confirms the work of Shuddemagen. This demagnetizing factor has been ignored in a great deal of the work which has been published, and should be corrected.

21. Magnetic Potential.—The preceding paragraphs have described the different ways in which magnetic fields may be produced. Various ways have been discussed for describing a magnetic field. There is still another very useful method by which the qualities of a magnetic field may be expressed which is called the potential of the field. Its significance may be brought out best by considering the work performed in moving a unit magnetic pole against the force of a magnetic field. This is a

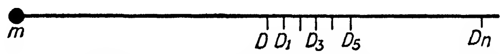


FIG. 38.—Magnetic potential in a magnetic field of force.

fictitious but helpful point of view. Suppose a magnetic field exists due to a positive magnetic pole m (Fig. 38). At any point situated at a distance D from the pole m, the force of repulsion on a unit pole of like sign is equal to m/D^2. At a distance D_n the force will be m/D_n^2 while at other intermediate points on the line connecting m and D_n the force will be m divided by the square of the distance between m and that point. If the unit positive pole is carried from D_1 to D it will be carried against the force of repulsion due to the positive pole m. The work performed will be the distance DD_1 multiplied by the force acting along the way. If the distance DD_1 is made very small, then the forces acting upon the unit positive pole at D and D_1 can be made so nearly the same that m/DD_1 may be taken as the average force acting along the path DD_1. Hence the work in moving the unit positive pole from D_1 to D is:

$$\begin{aligned}\text{Work} &= \text{Force} \times \text{distance}\\ &= \frac{m}{DD_1} \times (D_1 - D)\\ &= m\left(\frac{1}{D} - \frac{1}{D_1}\right). \qquad (51)\end{aligned}$$

[1] duBois, *Wiedemann Ann.*, **46**, 485, 1892.
[2] Ewing, *Philos. Trans.*, **176**, 535, 1885.
[3] Tanakadate, *Philos. Mag.*, **26**, 450, 1888.
[4] Mann, *Phys. Rev.*, **3**, 359, 1896.
[5] Wurschmidt, *Zeitsch. für Phys.*, **19**, 388, 1923.

In a similar fashion the work in moving the unit positive pole over the successive small intervals, into which the path between D and D_n has been broken, may be obtained. The total amount of work accomplished will be the sum of all the elements of work along the way.

$$\text{Work, } D_1 \text{ to } D = m\left(\frac{1}{D} - \frac{1}{D_1}\right)$$

$$\text{Work, } D_2 \text{ to } D_1 = m\left(\frac{1}{D_1} - \frac{1}{D_2}\right)$$

$$\text{Work, } D_3 \text{ to } D_2 = m\left(\frac{1}{D_2} - \frac{1}{D_3}\right)$$

$$\cdots\cdots\cdots \qquad \cdots\cdots\cdots$$

$$\cdots\cdots\cdots \qquad \cdots\cdots\cdots$$

$$\text{Work, } D_n \text{ to } D_{n-1} = m\left(\frac{1}{D_{n-1}} - \frac{1}{D_n}\right).$$

In taking the sum of all of these units of work, it will be noticed that all intermediate terms drop out, and

$$\text{Work, } D_n \text{ to } D = m\left(\frac{1}{D} - \frac{1}{D_n}\right). \tag{52}$$

If D_n is at an infinite distance, then the work in bringing a unit positive pole from infinity to the point D is:

$$\text{Work, } D_n \text{ to } D = \frac{m}{D}. \tag{53}$$

On its arrival at D the unit magnetic pole, by virtue of its position in the field of force, possesses the ability to do work. In other words, the unit magnetic pole possesses potential energy at the point D equal to m/D. This is spoken of as the *potential of the magnetic field* at the point D. It is equal to the amount of work done in bringing a unit positive magnetic pole from an infinite distance to that point. The force at the point D is m/D^2, while the potential is m/D.

The rate at which the potential varies with respect to the distance along the line D_n to D will be $\frac{\text{Potential}}{\text{Distance}}$. This *space rate of change of potential* is equal to the field strength. Expressed symbolically,

$$\frac{m/D}{D} = \frac{m}{D^2} = \text{force at the point } D.$$

This equation furnishes another definition: *Potential may be defined as a quantity whose rate of variation in any direction is the strength of the field in that direction.* This applies to any field of force whether it be gravitational, electrical, or magnetic. Turning this definition over once more we may say: *The field strength at any point in a field of force is the potential gradient of the field at that point.*

22. Magnetic Potential Difference.—The work performed in carrying the unit positive magnetic pole from one point to another

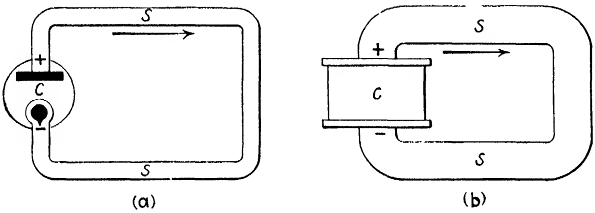


FIG. 39.—A striking analogy exists between the electric current developed by an electromotive force and the magnetic flux developed by a magnetomotive force.

was given in the preceding paragraph. This amount of work is the potential difference between the two points. There is a very good analogy between electric and magnetic potential differences. The potential difference between the poles of an electric cell is equivalent to the work expended in carrying a unit positive charge from the negative to the positive electrode. This potential difference is called the voltage or the electromotive force of the cell. In Fig. 39*a* is a conductor connected to a cell. The electromotive force will cause a current to flow in the conductor. In order to carry a unit positive charge around the circuit against the electric field, a certain amount of work will be required which is called the electromotive force of the cell. In Fig. 39*b* let *SS* be an iron link on one limb of which a coil *C* is wound. When a current flows in the coil a magnetic flux is produced around the magnetic circuit. In order to carry a unit magnetic pole around the circuit against the magnetic

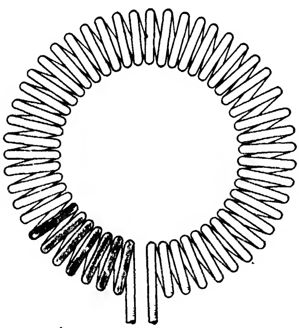


FIG. 40.—A solenoid bent into a circle is called a toroid. If an iron core filled the solenoid it would be called a toroidal electromagnet.

field, a certain amount of work will be required, which by analogy to the cell will be called the magnetomotive force of the coil. This work is easily calculated for a toroid. Figure 40 shows a simple *toroid.* Let L be the average of the inner and outer circumferences. According to Eq. (35), the force acting inside the windings of the toroid will be:

$$H = \frac{4\pi NI}{L}. \tag{54}$$

$$\begin{aligned} \text{Work} &= \text{force} \times \text{distance} \\ &= \frac{4\pi NI}{L} \times L = 4\pi NI = HL. \end{aligned} \tag{55}$$

This is the magnetomotive force of the toroid. The centimeter-gram-second unit of magnetomotive force as adopted in some countries is the *gilbert.* If I is measured in amperes, we have,

$$\text{mmf} = 0.4\pi NI \text{ gilberts.}$$

The gilbert is the magnetomotive force produced by a current of $10/4\pi$ amperes flowing in a single turn of wire. The product NI in Eq. (54) is called the *ampere-turns* of the toroid. A magnetomotive force expressed in gilberts is reduced to the equivalent number of ampere-turns by dividing by $4\pi/10$. The magnetomotive force around any magnetic circuit is equal to $4\pi/10$ times the number of ampere-turns linked with the magnetic circuit. The question as to whether the gilbert or the ampere-turn should be employed as the unit of magnetomotive force is a vexing one. To the author B and H are not different physical quantities. If a coil were concealed inside of a box it would be impossible from the quality of the magnetic field outside to distinguish between a coil with an iron core and one without the iron. To say that "the natural unit for the magnetomotive force is the ampere-turn," is getting the cart before the horse. What is one to do with the magnetomotive force of the field between the poles of a lodestone? 4π is a factor which ought not to be confusing to any student who has had an introduction to the idea of magnetic flux. Far more confusing is it to mix concepts and that is what one is doing in saying that B and H are different physical quantities. If B and H have different physical dimensions, let us stop a part of the confusion by expressing one in gausses and the other in something else. From Eq. (55) there follows the relation:

$$\text{Gausses} = \frac{\text{gilberts}}{\text{centimeters}}.$$

The field intensity of a solenoid may be spoken of as so many gilberts per centimeter just as the electric field intensity is frequently referred to as so many volts per centimeter. In general, one may say that a magnetomotive force is that which produces or tends to produce a magnetic flux.

23. Reluctance.—The total magnetic flux through an iron core filling a long solenoid is:

$$\phi = BA = \mu HA = \frac{4\pi NIA\mu}{L}$$

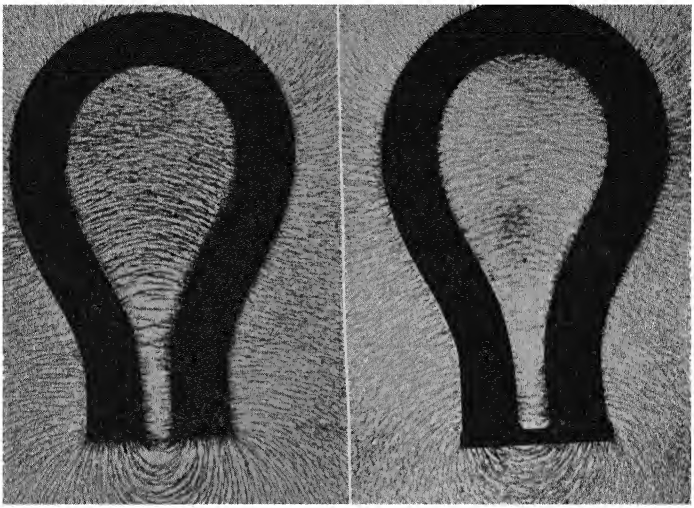
$$= \frac{4\pi NI}{\frac{L}{A} \times \frac{1}{\mu}}$$

$$= \frac{\text{mmf}}{R}, \tag{56}$$

where R is equated to the value, $L/A \times 1/\mu$. R is called the *reluctance* of the magnetic circuit. It is that property of the circuit which resists the penetration of the lines of force. Bosanquet[1] pointed out that Eq. (56) is analogous to Ohm's law. The analogy is apparent only in form. It breaks down at the start by assuming that the magnetic circuit is perfect. No such circuit exists for magnetic flux as prevails for electric currents. No matter how complex the circuit, the electric current remains constant throughout the entire length of the conductor. In the case of a magnetic circuit, leakage of the lines of flux occurs all along the magnetic conductor. In Fig. 41 is shown the field of a permanent horseshoe magnet. If it were a perfect magnetic circuit, all of the lines of force would emerge from one pole and pass over to the other. It will be noted that there is leakage all along the surface of the magnet. It is somewhat diminished when a keeper is placed across the poles as shown in Fig. 42. Even in this condition the magnetic circuit is not perfect. If Bosanquet's law should be applied to a composite magnetic circuit, it must make the very great assumption that the flux remains constant all along the circuit. In general, however, the greater the uniformity of reluctance along the path of the magnetic lines of induction, the less will be the leakage. The toroidal electromagnet makes an excellent example of this fact. There are no definite laws governing magnetic leakage. In planning a

[1] BOSANQUET, *Philos. Mag.*, **15**, 205, 1883.

new dynamo or motor, it is possible, empirically, to determine approximately the loss in lines of induction by leakage. Once the dynamo or magnet is constructed it is possible to ascertain, with considerable accuracy, the loss by leakage. In the case of a generator or a motor it is important that there be as little leakage as possible. Other things being equal, the most efficient machine is the one putting as large a magnetic flux as possible across the gap where the armature rotates. Thinking in terms



FIGS. 41 and 42.—The magnetic field about a horseshoe magnet without and with a keeper.

of such a machine we may define the magnetic leakage as the "difference between the total number of lines of induction produced by the magnetomotive force and the number that are useful in generating an electromotive force." The percentage loss of lines of induction may be calculated by the formula,

$$p = 100\frac{\Theta}{\phi},$$

where ϕ is the total number of magnetic lines, and Θ is the total number of stray lines of induction.

It is customary, however, to proceed on the basis that the leakage is small, or at least can be accounted for, and apply

Bosanquet's law to the average magnetic circuit. R in Eq. (56) may be written

$$R = \rho\frac{L}{A}, \tag{57}$$

where ρ is the *specific reluctance* or *reluctivity* of the material.

$$\rho = \frac{1}{\mu},$$

i.e., the reluctivity of a substance is the reciprocal of its permeability. In form the above relationships are identical with the relations between resistance and specific resistance, conductivity and resistivity in electric current phenomena.

The unit of reluctance is the oersted. It is defined as the reluctance of a circuit which requires a magnetomotive force of one gilbert to establish a flux of one maxwell. Equation (57) is the defining equation for the unit of reluctance, *viz.*, that the oersted is the reluctance of an air gap, one centimeter long and one square centimeter in cross-section. From (56),

$$R = \frac{\text{mmf}}{\phi}$$

$$\text{Oersteds} = \frac{\text{gilberts}}{\text{maxwells}}. \tag{58}$$

Following out the idea of the analogy to Ohm's law, the oersted of reluctance corresponds to the ohm of resistance. Conductance and resistance are reciprocal quantities as are reluctance and permeance, all being characteristic of their corresponding circuits. Resistivity and conductivity for electricity are like reluctivity and permeability for magnetism, characteristics of the material of the conductors.

There is still another sense in which the analogy between Ohm's law for electricity and Bosanquet's for magnetism is not complete. Resistance is independent of current strength while reluctance varies with magnetic flux which, in turn, depends on the permeability. The continuous flow of electricity through a conductor produces heat. No energy is required to maintain a continuous magnetic flux, although work is performed in establishing or reducing the flux through a ferromagnetic body. This means that there is no analogy to Joule's law in the magnetic circuit.

In applying Bosanquet's law to a composite magnetic circuit, one proceeds in the same way as for Ohm's law. The total

magnetomotive force to establish the desired flux across the various portions of the circuit is given by the expression,

$$\Sigma\text{mmf} = \phi R = \phi R_1 + \phi R_2 + \phi R_3 + \cdots, \tag{59}$$

where R_1, R_2, and R_3 are the reluctances of the various elements of the magnetic circuit. Since

$$\Sigma\text{mmf} = (\text{mmf})_1 + (\text{mmf})_2 + (\text{mmf})_3 + \cdots$$

and

$$R = \frac{L_1}{A_1}\frac{1}{\mu_1} + \frac{L_2}{A_2}\frac{1}{\mu_2} + \frac{L_3}{A_3}\frac{1}{\mu_3} + \cdots,$$

it follows that

$$\phi = \frac{\Sigma\text{mmf}}{\frac{L_1}{A_1}\frac{1}{\mu_1} + \frac{L_2}{A_2}\frac{1}{\mu_2} + \frac{L_3}{A_3}\frac{1}{\mu^3} + \cdots}. \tag{60}$$

24. Maxwell's Line Integral.—The work performed in carrying a unit positive pole from one point to another in a magnetic field was called the magnetic potential difference between the points. The work of carrying the unit pole around a complete magnetic circuit was called by Bosanquet the magnetomotive force (mmf) of the magnetic circuit. Maxwell called this expression the *line integral of the magnetic force.* A simple case is the line integral of the magnetic force at a distance r from the center of a long straight conductor. At a distance $r > R$, Fig. 43,

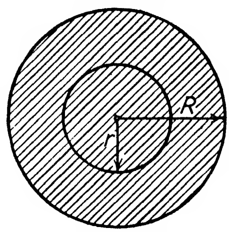


FIG. 43.—No magnetic field exists within a hollow conductor due to a current flowing in it.

$$H = \frac{2I}{10r} \text{ gausses if } I \text{ is in amperes.}$$

To carry a unit positive magnetic pole around such a conductor against the direction of the circular field would be:

$$\begin{aligned} \text{Work} &= \text{force} \times \text{distance} \\ &= \frac{2I}{10r} \times 2\pi r = \frac{4\pi I}{10} \text{ ergs.} \end{aligned} \tag{61}$$

For any number of wires N the line integral will be $4\pi NI$, which is the magnetomotive force of a solenoid. *The line integral of the magnetizing force about a closed magnetic circuit is equal to* 1.2566 *multiplied by the ampere turns linked with the magnetic circuit.* If the path along which the integral is taken is a closed

one and does not link with any current, the line integral is zero because in (61) I will be equal to zero. The line integral along a path inclosing unit area is sometimes called the curl of the field.

$$\text{Curl } H = 4\pi(\text{current density}). \tag{62}$$

Maxwell's line integral furnishes a very easy way in which to study the distribution of the magnetic field about an electric current flowing in a conductor. If the current density is uniform, this line integral may be used to find the field strength at points not only without but also within the conductor. Let R (Fig. 43) be the radius of the conductor. At a point $r > R$ the line integral for the field at a distance r from the center of the wire is:

$$\int_0^{2\pi r} HdL = 2\pi r H_0, \tag{63}$$

but $4\pi I$ is also the line integral, and so

$$2\pi r H_0 = 4\pi I$$

$$H_0 = \frac{2I}{r} \qquad \text{(21) and (64)}$$

$$H_0 \propto \frac{1}{r}.$$

For a point $r < R$,

$$\int_0^{2\pi r} HdL = 2\pi r H_i.$$

Remembering that there is no magnetic effect inside of a hollow conductor carrying an electric current, there will be no effect at r due to any current outside of that point, so that

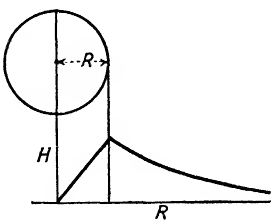


FIG. 44.—A curve showing how the magnetic field varies inside and out of a conductor, as the distance R from the center increases.

$$2\pi r H_i = 4\pi \frac{r^2}{R^2} I$$

$$H_i = \frac{2rI}{R^2} \tag{65}$$

$$\therefore H_i \propto r.$$

From (64) and (65) it follows that the field strength anywhere inside of a conductor varies as the distance from the center, while for points outside, the field varies inversely as r. At the surface, $r = R$, and the two Eqs. (64) and (65) become equal. Figure 44 gives a graph of the variation of the field as one goes from the center of the conductor to any point outside (see Sec. **14**).

25. Magnetic Flux in the Core of a Toroid.—Figure 20 shows that inside the ring the number of turns per unit length is not the same as at the outside. The distribution of magnetic flux in the iron core of a toroid, therefore, will not be so simple as for the bar in a straight solenoid. But because the toroid is so important commercially, approximations will be made and the total flux in the core calculated.

The line integral about the endless solenoid in Fig. 40 at a distance X from the center is, when no iron is present in the coil,

$$2\pi X H_x = 4\pi N I$$

$$H_x = \frac{2NI}{X}.$$

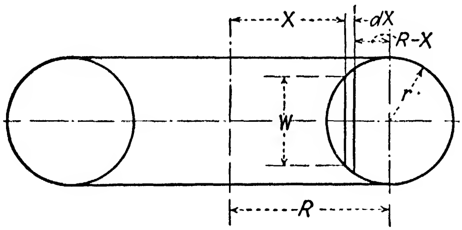


FIG. 45.—The cross-section of the core of a toroidal electromagnet.

If iron is present,

$$B_x = \frac{2\mu NI}{X}. \tag{66}$$

For a large thin ring no serious error will be introduced if it is assumed that there is a uniform flux around the ring at an average distance R. The total flux will be:

$$\phi = \frac{2\mu NIA}{R} = \frac{4\pi NI}{\frac{L}{A}\frac{1}{\mu}}. \tag{67}$$

For a core whose cross-section of limb is large, this approximation cannot be used. Figure 45 gives the cross-section of such a core: r is the radius of the cross-section of the limb, while R_1 and R_2 (not shown) are the inner and outer radii of the ring respectively. Let a thin strip be thought of as being cut out around the ring whose width is $W = 2\sqrt{r^2 - (R - X)^2}$ and whose thickness is dX. Its area will be:

$$A = 2\sqrt{r^2 - (R - X)^2}dX. \tag{68}$$

Since at the distance X,

$$B_x = \frac{2\mu NI}{X},$$

the total flux through the section thus cut out is:

$$d\phi = \frac{4\mu NI}{X}\sqrt{r^2 - (R - X)^2}dX. \tag{69}$$

The total flux through the ring is obtained by integrating between the limits, R_1 and R_2. Only as μ is taken as a constant can this be done, and even then it is somewhat involved,[1] but when completed,

$$\begin{aligned}\phi &= 4\mu NI \int_{R_1}^{R_2} \sqrt{\frac{r^2 - (R - X)^2}{X^2}}\,dX \\ &= 4\pi\mu NI(R - \sqrt{R^2 - r^2}).\end{aligned} \tag{70}$$

26. Energy in Magnetic Field per Unit Volume.—When any space is magnetized, there is stored up in that space a certain amount of energy, which may be expressed as the energy per unit volume. To make the case more general, it is assumed that the space magnetized has a permeability of μ. In a unit cube of the substance there will be developed on the ends of the cube normal to the lines of induction a polarity m such that, $4\pi m = \mu H = B$, the flux per unit area; or

$$m = \frac{\mu H}{4\pi}. \tag{71}$$

The magnetization of a cube of any substance means the polarization of the medium or the same as bringing magnetic poles of strength m from an infinite distance up to the point considered. The energy, therefore, will be:

$$E = \tfrac{1}{2}mV,$$

where V is the potential of the point to which m is brought.

$$E = \frac{\tfrac{1}{2}\mu HV}{4\pi} = \frac{\mu HV}{8\pi}, \tag{72}$$

but

$V = Hd$, and since $d = 1$ in the unit cube discussed.

$$E = \frac{\mu H^2}{8\pi} \text{ ergs}, \tag{73}$$

the energy per unit volume.[2]

[1] TIMBIE and BUSH, "Principles of Electrical Engineering," pp. 218–224; see PIERCE, "Table of Integrals," 160, 183, and 187; LLOYD, *Sci. Papers* Bur. Stand., **5**, 442, 1908.

[2] WALL, "Applied Magnetism," pp. 41–43.

It is a general principle of dynamics that any body, free to move in a field of force, moves in such a way as to reduce the potential energy of the system to a minimum. A falling stone in the field of gravity illustrates this principle. When a body is brought into a magnetic field it becomes magnetized by induction and, therefore, possesses energy per unit volume according to Eq. (73). This is potential energy and if the body is free to move, it will move so as to reduce this energy to a minimum. To put Eq. (73) into a form that may be interpreted, let the body have a volume $W = AL$. The total energy of the body will then be:

$$E = \frac{\mu H^2 AL}{8\pi}. \tag{74}$$

The number of lines of induction through the ends of any cylinder thus magnetized is:

$$\phi = \mu HA$$

$$H = \frac{\phi}{\mu A}. \tag{75}$$

Eliminating H from (75)

$$E = \frac{\mu\phi^2 AL}{8\pi\mu^2 A^2} = \frac{\phi^2 L}{8\pi\mu A}. \tag{76}$$

If $R = L/\mu A$, the reluctance,

$$E = \frac{\phi^2 R}{8\pi}. \tag{77}$$

If this last expression represents the energy of our magnetized cylinder, and the energy of the same is to diminish when it moves freely in space, then one way in which this can happen is by R becoming smaller. This may be accomplished in the case of solids by turning, if the body has different dimensions in different directions. In the case of liquids, they will deform their shape (Figs. 13 and 14). If the motion is not constrained, the bodies possessing a permeability factor greater than unity will move toward the position of greatest field intensity. Diamagnetic bodies will move the other way.[1]

27. Induction and Hysteresis Curves.—In the process of magnetizing a steel rod by means of a solenoid, the flux density through the steel does not proceed by equal steps with the magnetizing force. The relation which does exist between the flux density B in the steel and the magnetizing force H, is shown

[1] THOMSON, "Papers on Electrostatics and Magnetism," p. 515, 1872.

in curve OM (Fig. 46). If, after attaining the point M on the curve (usually designated as B_{max}), the magnetizing force is slowly decreased, the flux density does not return along the same curve as when H was being increased but lags and follows the path M to R. At the point R the magnetizing current in the solenoid has been reduced to zero, although the steel rod still retains some magnetic activity. This amount of magnetization OR, which the rod retains, is called the *residual magnetism* of the steel. The property of retaining to a greater or less degree a certain amount of magnetization is called the *retentivity* of the substance. The terminology of magnetism is rather confusing regarding some of these terms. Several modern writers speak of the value of OR as the remanent magnetism. The consensus of opinion among magneticians at present is to reserve the term *remanent magnetism* for the open-circuit residual magnetism as in the case of a U-shaped permanent magnet with the keeper off. The magnetization through the arms of a U-shaped magnet is different when the keeper is off than when it is on (see Figs. 41 and 42). The value of keepers for permanent magnets lies in their power to reduce demagnetization effects. The magnetism retained in an iron ring, after the magnetizing force is with-drawn, is an illustration of residual magnetism. There are no free poles left to further reduce the lines of magnetization.

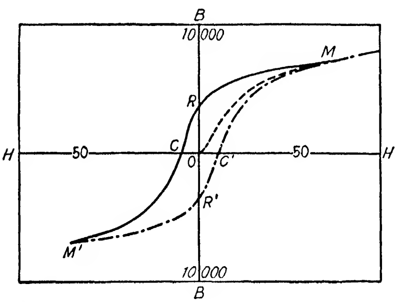


FIG. 46.—Induction and hysteresis curve for a ferromagnetic body.

In order to reduce the magnetization to zero a magnetizing force in the opposite direction must be applied to the specimen. The value of $-H$, at which B is reduced to zero, is called the *coercive force*. This is measured by the magnitude of OC in Fig. 46. Increasing the magnetizing force in the same direction as OC carries the curve to M'. This point corresponds to M on the other limb of the curve. In an exactly analogous process the curve expressing the relation between B and H may be carried from M' to M as was employed in going from M to M'. OR' gives the value of the residual magnetism as did OR, and correspondingly OC' measures the coercive force.

The curve OM (Fig. 46) is called a *normal induction curve* and the loop $MRCM'R'C'M'$ is known as a *hysteresis loop*. A knowledge of the forms of these curves for various ferromagnetic bodies gives us a very good idea of the fitness of such materials for various electromagnetic devices. *Hysteresis* is a term applied to the tendency of magnetic materials to persist in any magnetic state which already exists. It is due to this property that we have such a thing as a permanent magnet. Good permanent magnets should have high permanence and great coercive force. Tungsten, chromium, molybdenum,[1] and cobalt[2] steels are among the best alloys for making permanent magnets. Honda[3] discovered a steel known as K. S. magnet steel which has proved to be very satisfactory for this purpose. The constancy of fine ammeters and voltmeters proves how permanent some of these steels remain, once they have been properly treated and aged.[4] It has been found that immediately after hardening, the coercive force of steel decreases very rapidly. At the end of several months this decrease in coercive force becomes quite small and, finally, the coercive force remains very constant. This decay of coercive force seems to be due to the breakdown of the stability of feebly oriented groups of elementary magnets and to metallurgical processes going on within the metal. By a process known as "aging," this decay of coercive force may be hastened and the point of stability attained in a comparatively short time. If left to natural processes it would take years to attain this same stability.

By studying the magnetization curves of various alloys, and in particular the effects on these curves when special heat treatments are applied to the metals, it has been possible to find alloys which will give certain specified magnetic properties. This is illustrated by the discovery of *permalloy*[5] which possesses a very high permeability for small magnetizing forces. Similarly, the discovery of *perminvar*[6] is another example of what can be accomplished along these lines for another special use. In this

[1] MOIR, *Philos. Mag.*, **28**, 738, 1914.

[2] MICHEL and VEYRET, *Rev. gén. élec.*, Jan. 12, 1924;
SANFORD, *Sci. Papers* Bur. Stand., **22**, 557, 1927.

[3] HONDA and SAITO, *Sci. Repts.* Tôhoku Imp. Univ., **9**, 417, 1920; WALL, "Applied Magnetism," p. 49, 1927.

[4] SPOONER, "Properties and Testing of Magnetic Materials," p. 136, 1927.

[5] ARNOLD and ELMEN, *Jour. Franklin Inst.*, **195**, 621, 1923.

[6] ELMEN, *Jour. Franklin Inst.*, **206**, 317, 1928.

alloy, "magnetic measurements indicated that up to moderate field strengths the permeability of this nickel, cobalt, and iron alloy was remarkably constant." At first glance it seems possible to get almost any property one wishes by means of proper alloys and heat treatments. Under this head may be mentioned again the need of a ferromagnetic material possessing high permeability in strong fields. This whole field of research in magnetic alloys is a fascinating one.

28. Energy Loss Due to Hysteresis.—When a solid piece of iron or other substance showing hysteresis is left in an alternating magnetic field it gradually warms up, indicating that energy is

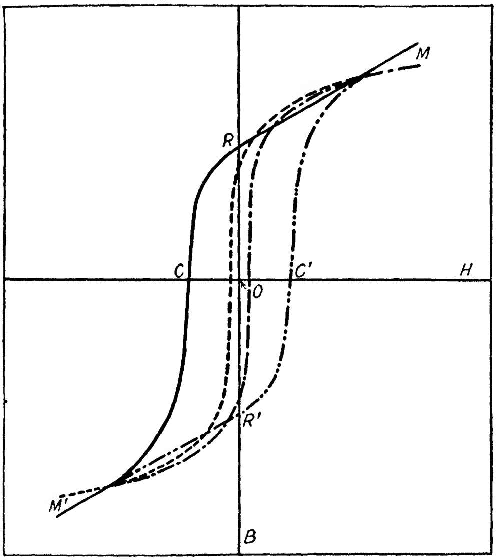


FIG. 47.—(*a*) The slim hysteresis loop is for soft annealed nickel. (*b*) The broad loop is for nickel hardened by cold rolling, 93 per cent cold reduction.

being dissipated. There are two causes for this loss of energy in a substance possessing hysteresis. One is due to the induced electric currents flowing through the magnetic medium and thereby producing heat according to the law of Joule. These eddy currents are sometimes called Foucault currents. The loss from this source is called *eddy-current loss.* The second cause of energy dissipation is due to the process of magnetization itself. It is evident from Fig. 46 that the energy required to magnetize a specimen is not entirely recoverable on removing the magnetizing force. The magnetization does not decrease

to zero for zero field but an oppositely directed force must be applied to bring the intensity of magnetization back to zero. This form of energy loss is called *hysteresis loss.* The eddy-current loss, combined with the hysteresis loss, is called the *core loss.*

In building transformers and other electric machinery the cores are usually built of laminated iron. These thin sheets of iron are more or less insulated and when placed normal to the direction of the induced electromotive force the eddy currents are thus practically eliminated. This leaves only the hysteretic losses to make up the core losses.

The reduction of hysteresis loss in building transformers and other electrical appliances becomes a first-class problem in magnetism. Lloyd[1] pointed out in 1910 that America alone had a financial loss of approximately ten million dollars annually, due to hysteresis loss. Here again proper alloys and proper heat treatment offer one avenue of approach to the problem.

It has been shown by Warburg[2] that the area of a hysteresis loop is a measure of the amount of hysteresis loss during a *magnetic cycle.* When this loss is small, the area of the loop will be small as shown in Fig. 47*a*. Figure 47*b* shows a loop for a material with a much higher hysteresis loss. In order to derive an expression for the energy loss due to hysteresis, suppose an iron ring of uniform cross-section A is wound as a toroidal electromagnet. Let n be the number of turns per unit length so that the total number will be Ln. E is the electromotive force applied to the terminals of the magnetizing coil. If the magnetizing current is changed by a very small increment dI in the time dt, then the induction B will also be changed by a small amount dB.

$$AB = \phi \tag{78}$$

is the total induction through the core. The change in the flux AB gives rise to a counter electromotive force E', against which the magnetizing current must flow and thus does work. If the total flux is linked with Ln turns, then

$$E' = Ln\frac{d\phi}{dt} = LnA\frac{dB}{dt}. \tag{79}$$

[1] LLOYD, *Jour. Franklin Inst.*, **170,** 1, 1910.

[2] WARBURG, *Freiburg. Ber.*, no. 8, vol. 1, p. 1, 1880; *Wiedemann Ann.*, **13,** 141, 1881.

Applying Ohm's law to the circuit of the coil,

$$E - E' = IR \tag{80}$$

$$E - LnA\frac{dB}{dt} = IR$$

$$Edt = LnAdB + IRdt$$

$$EIdt = ILnAdB + I^2Rdt. \tag{81}$$

The left-hand member is the energy delivered to the magnetizing coil in the time dt. The second term on the right-hand side of the equation is the energy which is lost as heat in the wire of the coil. The other term is the energy dissipated in the change of B by the amount dB.

$$W = ILnAdB. \tag{82}$$

This equation shows that the energy thus dissipated is quite independent of the time, no matter what the process of magnetization. If the process is cyclic, a part of the energy is irrecoverable. In other words, the energy lost in magnetizing a bar of iron through a complete cycle cannot be augmented or decreased by varying the frequency of the alternation. The energy used in the process described in (82) may be expressed in terms of unit volume since LA is the volume of the iron core. Thus,

$$\frac{dW}{dV} = W_v = nIdB. \tag{83}$$

The magnetizing force in the coil is

$$H = 4\pi nI \text{ gausses} \tag{35}$$

$$nI = \frac{H}{4\pi}.$$

Therefore,

$$W_v = \frac{1}{4\pi}\int HdB \tag{84}$$

$$= \text{ergs/ccm/cycle,}$$

if the process of magnetization includes a complete cycle. This is the form in which Hopkinson[1] developed the expression for hysteresis loss. Warburg's expression was

$$W_v = \int Hd\mathfrak{I} \tag{85}$$

$$= \text{ergs/ccm/cycle.}$$

It is at once evident that these expressions are interchangeable. If the magnetic process is cyclic and B and H have the same

[1] HOPKINSON, *Philos. Trans.*, **176**, 466, 1885.

values at the end as at the beginning of the cycle, we can go from one form to the other.

$$B = H + 4\pi \mathcal{I},$$

and, therefore,

$$HdB = HdH + 4\pi Hd\mathcal{I}$$

$$\int HdB = \int HdH + 4\pi \int Hd\mathcal{I}.$$

When H passes through either the process $+H_{max}$, $-H_{max}$ and back to $+H_{max}$, or from $+H_{max}$ to $-H_{max}$, then $\int HdH = 0$. From these conditions emerges Warburg's expression, $\int Hd\mathcal{I}$

$$W_v = \int Hd\mathcal{I} = \frac{1}{4\pi}\int HdB. \tag{86}$$

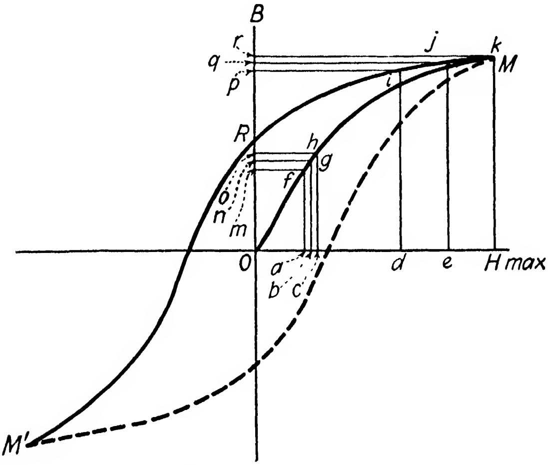


FIG. 48.—Hysteresis loop. Recoverable and irrecoverable energy indicated.

It is very helpful to interpret Eq. (84) in terms of a hysteresis curve as shown in Fig. 48. If the field H is increased from Oa to Ob (an infinitesimal amount), the value of B will be increased by the small amount dB equal to $On - Om$,

$$\frac{Ob(On - Om)}{4\pi} = \frac{HdB}{4\pi}. \tag{87}$$

Thus the area of the small rectangle $mngf$ divided by 4π represents the work in changing B by the small amount dB. Similarly, in making the next step in H, the rectangle $nohg$ divided by 4π is the energy expended in the next magnetization process. By increasing the field to H_{max} the work done in changing the flux is the sum of all the little rectangles which are thus successively

built up, or the total area of OrM. On reducing H back to zero the induction does not follow along the old curve OM but goes back to R. In this process energy is given up by the system. Changing H_{max} to H_e produces the change in B represented by the distance $Or - Oq$,

$$\frac{H_{\text{max}}(Or - Oq)}{4\pi} = \frac{HdB}{4\pi}, \tag{88}$$

which is the area of the small rectangle $qrkj$ divided by 4π. This represents the energy delivered back again by reducing the field H in this process. The total energy recovered will be represented by the sum of all the small rectangles thus successively built up. The energy recovered in reducing H from H_{max} to zero will be represented by the area RrM. The irrecoverable energy will be graphically represented by the area $OrM - RrM$,

$$OrM - RrM = ORM. \tag{89}$$

Continuing this process, all around the hysteresis loop, will show that the total irrecoverable energy is:

$$W_v = \frac{[\text{area of the loop}]}{4\pi} \text{ ergs/ccm/cycle} \tag{90}$$

$$= \frac{HdB}{4\pi} \text{ ergs/ccm/cycle.}$$

This loss for any ferromagnetic material may be determined by producing a hysteresis curve and getting its area. For convenience a graph of the loop is made on metric cross-sectional paper. The values of B are always larger than H and cannot be plotted to the same scale. If x represents the number of H units per division on the metric paper and y the corresponding number of units per division along the B axis, then

$$W_v = \frac{xy}{4\pi} [\text{area of loop in sqcm}] \text{ ergs/ccm/cycle.} \tag{91}$$

The area in square centimeters is very conveniently determined by means of a *planimeter*. This loss in energy due to a process of magnetization may also be expressed in watts per pound or in watts per kilogram. Finding the hysteresis loss by means of the area of a hysteresis loop is tedious. Searle[1] has given a very neat method for finding the hysteresis loss directly by experi-

[1] Searle, *Proc. Camb. Philos. Soc.*, **9**, 2, 1895;
Searle and Bedford, *Philos. Trans.*, **198**, 33, 1902.

mental means. Searle's whole treatment of this subject should be studied with care.

A piece of ferromagnetic material must be put into a cyclic state before the hysteresis curve will form a closed loop. If a piece of iron which has never been magnetized is put through its first cycle the loop will not close at M (Fig. 46). Quite a number of complete cycles must be imposed on the substance, say twenty to thirty at least, before it is in a fair cyclic condition. Tapping[1] the iron with quick sharp blows while the iron is under the influence of the magnetizing force will help this process.

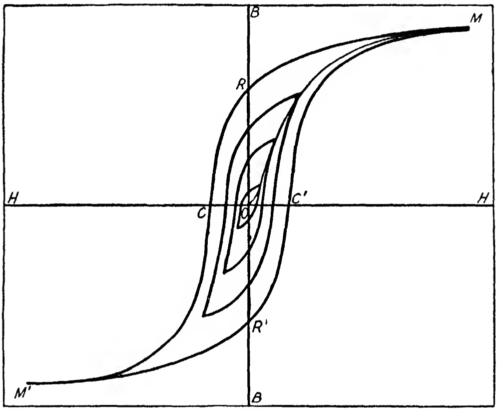


FIG. 49.—Graded cyclic magnetization of iron. The normal induction curve lies along the peaks of the gradually decreasing hysteresis loops.

In Fig. 49 is shown a series of hysteresis loops of successively smaller and smaller values of $H_{\max}$. In such a series it will be found that a normal induction curve is the locus of the cusps of the gradually decreasing hysteresis loops. As will be shown later, this is very important in the process of demagnetization by reversals with gradually decreasing fields.

29. The Steinmetz Constant.—If from Eq. (91), one computes the energy loss in ergs per cubic centimeter per cycle for different values of $B_{\max}$ and plots these values against the corresponding values of $B_{\max}$, a curve similar to Fig. 50 will be obtained. Steinmetz[2] expressed this relationship by an empirical equation of the form,

$$W_v = \eta B^k{}_{\max}, \tag{92}$$

[1] KELVIN, "Mathematical and Physical Papers," vol. V, p. 416.

[2] STEINMETZ, *Electrician*, **26**, 261, 1891; **28**, 425, 1892.

where k was given the value 1.6,

i.e.,

$$W_h = \eta B^{1.6}_{\max}. \tag{93}$$

Later, experimental work showed that this equation was not rigorous. It was found that k varied considerably from 1.6 at high- and low-flux densities. The equation fits the experimental data best between the flux densities of 1,500 to 12,000 gausses. η is called the *hysteretic constant* or *coefficient of hysteresis loss*, which varies from one ferromagnetic body to another. It is a characteristic of the material. In some of the best silicon steels it is as low as 0.0006 and in some of the tungsten steels it has a value of 0.058 (see Table II in the Appendix).

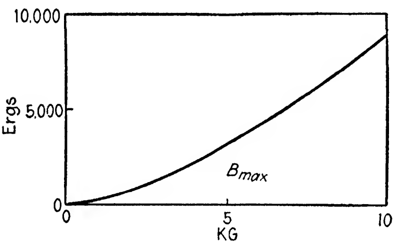


FIG. 50.—Type of curve from which Steinmetz derived his empirical equation for hysteresis loss.

This law of Steinmetz has been extended to include eddy-current losses as well. For this purpose the equation of energy for the core loss takes the form,

$$W_c = W_h + W_e = \eta f B^{1.6}_{\max} + E(f_x f B_{\text{mxa}} t)^2 \text{ ergs/ccm/sec.} \tag{94}$$

f is the frequency of alternation, E is the *eddy-current constant* which is a function of the specific resistance of the core, f_x is the *form factor* of the alternating wave of magnetic flux, η is the hysteresis constant and t is the thickness of the laminations. This form of expression for the core loss is of great importance in the commercial fields. Wall[1] has given a discussion of the methods to be employed in separating the hysteresis loss from the eddy-current loss in iron laminations.

Because the production, distribution, and control of electrical energy is so dependent upon a correct knowledge of the laws representing the relation between the electric current and magnetism, much labor has been spent in attempting to formulate a general law which shall express B as some function of H.

$$B = F(H). \tag{95}$$

Frölich[2] gave as a law the following expression:

$$B = \frac{H}{\alpha + \sigma H}. \tag{96}$$

[1] WALL, "Applied Magnetism," p. 221, 1927.

[2] FRÖLICH, *Electrotech. Zeitsch.*, pp. 141, 170, 1881; p. 73, 1882; p. 164, 1886.

Kennelly[1] introduced the *reluctivity* idea into Frölich's equation which gives it the form,

$$\rho = \alpha + \sigma H. \tag{97}$$

This has been called Kennelly's law, but it was first formulated by Fleming.[2] Gokhale[3] has discussed all of these laws and proposed one of his own. All labor under the same handicap that the constants have no physical significance. This has been pointed out by Sanford.[4] To quote from Sanford; "The problem of the correlation between the magnetic and other physical properties would be much simplified if a formula were available for expressing the magnetic properties in terms of constants having definite physical significance." This is a field sufficiently important to enlist further investigation.[5]

30. Demagnetizing Processes.—Once a bar of iron has been magnetized its past history has been altered so far as further experiments are concerned. There is no return to its *virgin state, i.e.*, a state in which the specimen has not been magnetized.

By processes which we call *demagnetization* a ferromagnetic specimen may be returned to a condition which may be duplicated time and again which we call the *neutral state*. In this state the specimen "yields as readily to a positive demagnetization force as to a negative one." Burrows[6], in a very thorough study of the best methods for demagnetizing iron in magnetic testing, offers the dictum that "the criterion of perfect demagnetization is that the induction shall be a maximum."

The *cyclic processes* shown in Fig. 49 give the clue to one process of demagnetization. If the piece of metal which has been magnetized is subjected to a progressively diminishing alternating field, the metal will eventually arrive at the neutral state. This is because the normal induction curve is the locus of the peaks of the gradually decreasing hysteresis loops. If the final loop is infinitely small, the normal induction curve will be at the point, $H = 0$, $B = 0$, which is the neutral point as defined. A ferromagnetic substance may be returned to this condition

[1] Kennelly, *Trans.* Amer. Inst. Elec. Eng., **8,** 485, 1891.

[2] Fleming, *Trans.* Amer. Inst. Elec. Eng., **3,** 569, 1886.

[3] Gokhale, Paper presented at meet. Amer. Inst. Elec. Eng., June 21–25, 1926.

[4] Sanford, *Sci. Papers* Bur. Stand., **21,** 743, 1927.

[5] Richter, *Electrotech. Zeitsch.*, **25,** 1241, 1910.

[6] Burrows, *Bull.* Bur. Stand., **4,** 212, 1907–1908.

repeatedly, but not to the virgin state. In the virgin state the first hysteresis loop is not a closed loop and therefore differs from the neutral state.

Summing up the evidence derived from his investigations Burrows concludes:

> The demagnetization should be accomplished by a current reversed at the rate of approximately one cycle per second, while gradually diminished in such a way that the rate of decrease of the induction is as nearly as may be uniform. An ammeter in circuit and a rough estimate of the shape of the *BH* curve will enable one to regulate the rate of decrease of current with sufficient exactness. The initial demagnetizing current should be sufficient to carry the induction beyond the knee of the *BH* curve and the final current should not be greater than the smallest magnetizing current to be used.

Steinhaus and Gumlich[1], in a very illuminating paper, give a picture as to the processes which go on in a ferromagnetic substance when going through the process of demagnetization. Searle[2] used an alternating current of 90 cycles from the city main and reduced this current to zero. The author has used a current of 60 cycles for demagnetizing purposes and finds if sufficiently high resistance can be thrown in gradually before finally being made infinite, that perfect demagnetization may be accomplished. The difficulty is that in the last stages of demagnetization the current must finally be broken, and the break will come with one phase or the other of the alternating current sufficiently large to give one last feeble magnetization in one direction or the other. The ideal condition for demagnetization is that the demagnetizing current should be *gradually* reduced to zero. There should be no sudden break at the last. A liquid potentiometer method for reducing the voltage to zero has been found to be very satisfactory.

Knott[3] in studying the interaction of circular and longitudinal magnetizations states that "Heating to a red-heat can alone truly demagnetize an iron wire." This method is eminently unsatisfactory for specimens in the ring form. To carry out such a method would necessitate winding and unwinding the turns put on the ring. In all probability, the physical properties

[1] Steinhaus and Gumlich, *Verh. der deutsch. phys. Gesellsch.*, **17**, 369, 1915.
[2] Searle, *Proc.* Inst. Elec. Eng., **34**, 61, 1904.
[3] Knott, *Philos. Mag.*, **30**, 244, 1890.

would also be changed by the heating process and a new specimen would result.

Mechanical tapping and shaking also help to demagnetize ferromagnetic bodies. Any sort of mechanical working[1] tends to accelerate the process of getting rid of the permanent magnetization.

31. Survey and Measurement of Magnetic Fields.—The preceding portions of this chapter have attempted to define and explain various terms and expressions used in describing a magnetic field. There will be an attempt in what follows to describe, with some detail, a few of the best methods employed in making magnetic measurements. If, as we are told, we know a thing to the extent we can measure it, then it is highly essential that the quantitative methods of measurement be discussed rather carefully.

a. Topographical Survey of a Field Surrounding a Magnet.—Figures 2 and 3, also Figs. 41 and 42, indicate very clearly that the magnetic field about a permanent magnet may be visualized by sprinkling iron filings in the immediate neighborhood of the magnet. This may be done best on a photographic plate. The plate is laid horizontally with the film side up and the magnet arranged symmetrically in the middle. Over the plate and magnet the filings are scattered as uniformly as possible. A small tea strainer works very well for sifting and scattering the filings. If the plate is gently tapped the filings will arrange themselves in a more orderly fashion. Then expose plate, magnet, and filings to the light of a 25-watt incandescent lamp for two or three seconds. Once a good negative is secured as many prints may be made as desired. The same process may be carried out with photographic paper, but one has only a single copy of a given distribution of the filings.

The mapping of a magnetic field may also be done by means of a small compass and needle (see Figs. 51 and 52). A bar magnet is laid on a large sheet of white paper with the axis of the magnet parallel to the earth's magnetic field, and left in this position until the map is completed. It will be of interest to make one map when the north pole of the magnet points north and another with the same pole pointing south. A small compass

[1] Robin, *Mem. trav. soc. ing. civ. France*, p. 798, 1912;
Moir, *Philos. Mag.*, **28**, 738, 1914;
Becknell, *Phys. Rev.*, **8**, 507, 1916;
Sherfer, *Electrician*, **87**, 263, 1921.

about $1\frac{1}{2}$ cm. in diameter is a convenient size. This is laid along side of the bar magnet and points are made with the pencil as near under the end of the compass needle as is possible. Placing

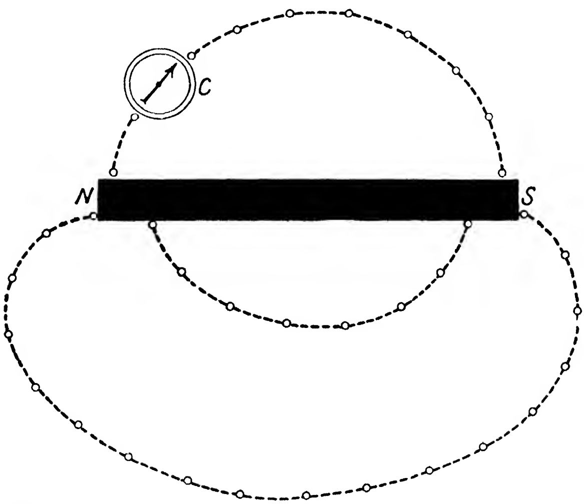


FIG. 51.—Mapping the magnetic lines of force about a magnet by means of a compass C and pencil.

the pencil point under the ends of the compass needle is facilitated by cutting small notches in the housing of the compass needle.

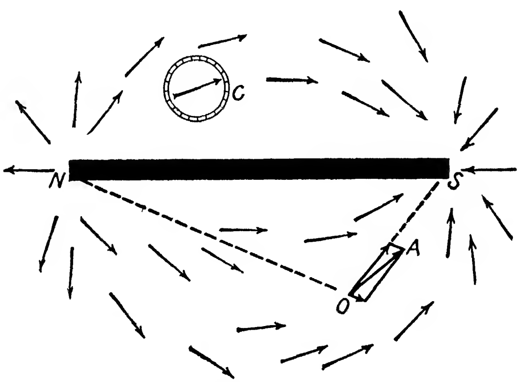


FIG. 52.—In plotting just the field of a magnet by means of a compass, the needle takes up a position parallel to the resultant of the two forces due to N and S.

For convenience these should be 180° apart. Next, the compass is moved away from the fixed magnet until the dot on the paper, formerly under the outer end of the compass needle, is brought

under the end nearest the bar magnet. If this process of extending the line of dots under the outer end of the needle is continued they become the loci of a curve extending from the positive to the negative pole. Again these lines indicate the paths along which the magnetic forces act. The lines may be drawn as thickly as is desired to give a thorough survey[1] of the field and visualize the directions in different parts of the field.

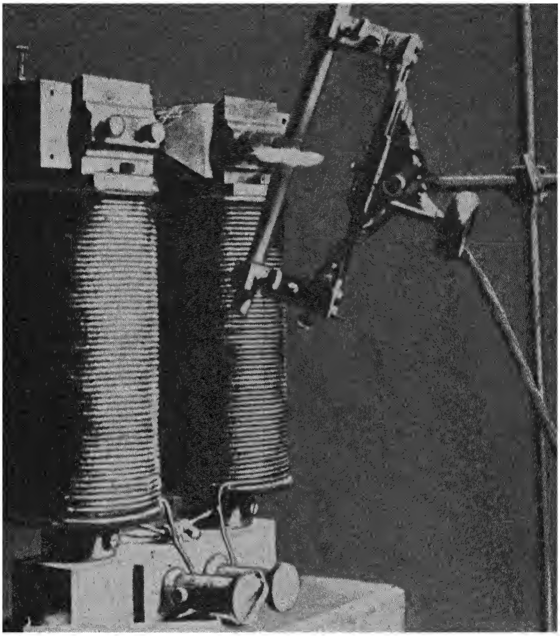


FIG. 53.—The alternating-current arc will be spread like a bat's wings when in a strong magnetic field. The wings at right angles to the field give the direction of the field and the length a rough measure of intensity.

Both for this method and for the iron filings it must be remembered that what one is actually mapping is the resultant field produced by those of the magnet and the earth. In a similar fashion the fields about solenoids, electromagnets, and other devices for producing magnetic fields may be studied by these methods. A rough survey of the field of a large electromagnet may be made by means of an alternating-current arc-light (Fig. 53). The effect of the magnetic field on the gas conductor of the

[1] GLAZEBROOK, "Electricity and Magnetism," p. 114, 1903;
PERKINS, "Electricity and Magnetism," p. 86, 1896.

arc is to push it first to one side and then to the other with each succeeding alternation. This spreads the arc normal to the direction of the magnetic field causing it to look like a bat's wings.

b. Field Intensity Determinations.—Magnetic field intensity measurements generally consist of a comparison of the unknown field with another field taken as a standard. This standard may be the earth's field whose value is obtained by an absolute method (Sec. 66), it may be the field of a solenoid evaluated by calculation as in Sec. **16**, or it may be some form of standard[1] magnetic field such as may now be obtained in the open market.

One simple but accurate method of comparison is to find the period of a freely swinging magnet, first in the standard and then in the unknown field. A magnetic needle suspended so as to swing freely either on a fiber or a very finely pointed needle is called a *magnetometer.* Such a needle, if displaced from its position of equilibrium in a magnetic field, will oscillate with a period given by Eq. (11),

$$t_s = 2\pi\sqrt{\frac{K}{MH_s}},$$

in which H_s is the strength of the known field, K the moment of inertia of the swinging magnet, and M its magnetic moment. Similarly for the unknown field,

$$t_x = 2\pi\sqrt{\frac{K}{MH_x}},$$

whence

$$\frac{t_s^2}{t_x^2} = \frac{H_x}{H_s}, \tag{102}$$

or

$$H_x = \frac{t_s^2}{t_x}H_s.$$

If H_s is given in gausses, H_x will also be expressed in gausses.

A coil connected to a ballistic galvanometer may often be substituted for a magnetometer in magnetic measurements. If such a coil, normal to the lines of force, is suddenly turned over in a magnetic field, a deflection of the galvanometer will occur which is proportional to the strength of the field.[2] By reversing the same coil, first in a known or standard field, and then

[1] STARLING, "Electricity and Magnetism," p. 262, 1924.
[2] WILLIAMS, *Jour. Franklin Inst.*, **182**, 353, 1916.

in an unknown, it is possible to determine the latter by the simple relation,

$$H_x = H_s \frac{d_x}{d_s}, \tag{103}$$

wherein d_s and d_x are the deflections of the galvanometer in the known and the unknown fields respectively. The deflections may be obtained either by rotating the coil or suddenly jerking it out of the field.

The magnetic field between the poles of an electromagnet is a very common one to be determined. Among the best methods for this purpose are those which depend upon the various effects developed in matter by a magnetic field. They will be described in the various chapters which discuss these effects. They may be merely indicated by name at this point: Bismuth spiral method,[1] Kerr optical methods,[2] Faraday optical methods,[3] electromagnetic methods,[4] and hydrostatic methods.[5] Kapitza[6] has developed a method for measuring very intense transient fields.

32. Determination of Magnetic Induction and Hysteresis.—For the purpose of building dynamos, transformers, and other electromagnetic devices, it is of the greatest importance that the permeability, hysteresis loss, coervice force, retentivity, and other magnetic properties of the material entering into their construction be thoroughly known. All of these properties may be obtained by determining the *normal induction* and *hysteresis* curves illustrated in Fig. 46. Instruments for obtaining these curves are called *permeameters*.

There are two general methods for determining the magnetic properties mentioned in the preceding paragraph, (1) the magnetometric method and (2) the ballistic method.

[1] Lenard and Howard, *Electrotech. Zeitsch.*, **9**, 340, 1888;
Lenard, *Wiedemann Ann.*, **39**, 619, 1890.

[2] Kerr, *Philos. Mag.*, **3**, 321, 1877; **5**, 161, 1878.

[3] Faraday, "Experimental Researches," vol. III, p. 1, 1855;
Verdet, *Ann. chim. phys.*, **41**, 370, 1854; **43**, 37, 1854; **44**, 1209, 1857; **52**, 129, 1858.

[4] Gans, *Physikal. Zeitsch.*, **8**, 523, 1907;
Paschen, *Physikal. Zeitsch.*, **6**, 371, 1905.

[5] Quincke, *Wiedemann Ann.*, **24**, 374, 1885;
du Bois, *Wiedemann Ann.*, **35**, 137, 1888; **51**, 549, 1894;
Williams, *Amer. Jour. Sci.*, **34**, 297, 1912.

[6] Kapitza, *Proc. Roy. Soc.*, **115**, 679, 1927.

a. Magnetometric Method.—This method depends upon the deflection of a magnetometer needle for the determination of the relation between $\mathcal{I}$ and H. It was developed by Ewing.[1] One essential to this method is that the *magnetometer needle* shall be located in a controlling field of known strength, H_c, which is usually the horizontal component of the earth's magnetic field.

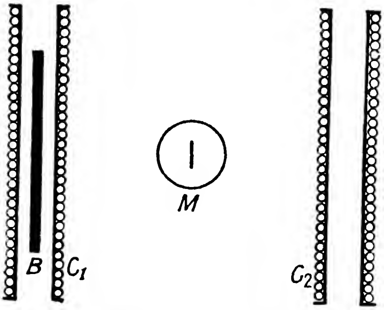


FIG. 54.—Symmetrical arrangement of coils for measuring magnetic properties by the magnetometric method. Coils in a horizontal position.

The author has always worked with one or the other of the compensated symmetrical systems shown in Figs. 54, 55, and 56. Two solenoids, exactly alike, are set up some distance apart with their axes parallel. They may be parallel in a horizontal position as in Fig. 54, their axes may be on the same straight line horizontally as in Fig. 55, or the coils may be set with their axes vertical and parallel as in Fig. 56. Whatever their arrangement the magnetometer is set symmetrically with respect to the two coils. Since the magnetometer needle will always be parallel to the horizontal component of the earth's magnetic field, the coils must always be placed with their axes in such a position that the forces due to the magnetized rod shall be normal to the field H_c. The two solenoids are electrically connected in series, with their fields just annulling each other at the magnetometer. Exact compensation

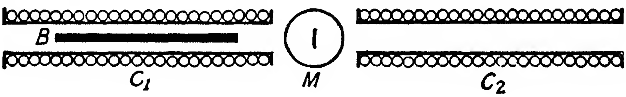


FIG. 55.—End-on arrangement of symmetrical coils for measuring magnetic properties by the magnetometric method.

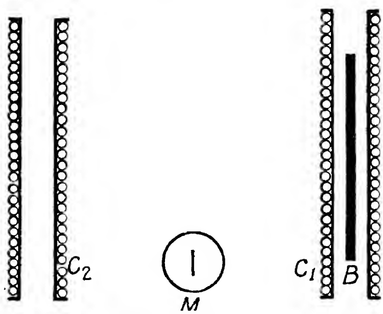


FIG. 56.—Arrangement of symmetrical coils in a vertical position for measuring magnetic properties by the magnetometric method.

[1] EWING, "Magnetic Induction in Iron and Other Metals," 1st ed., p. 37.

is effected by small movements of the coils. Compensation for one current strength should give compensation for all others.

If a rod B of ferromagnetic material is placed in one of the solenoids, magnetization occurs when an electric current is passed around the two coils. The induced polarity of the rod gives rise to a field uncompensated at the magnetometer, and so a deflection occurs. This deflection is a function of the intensity of magnetization. The total flux through the rod is:

$$B = H + 4\pi\mathfrak{I}.$$

Since H is compensated, $4\pi\mathfrak{I}$ is the part of the flux uncompensated, and therefore the deflection must be proportional to $\mathfrak{I}$.

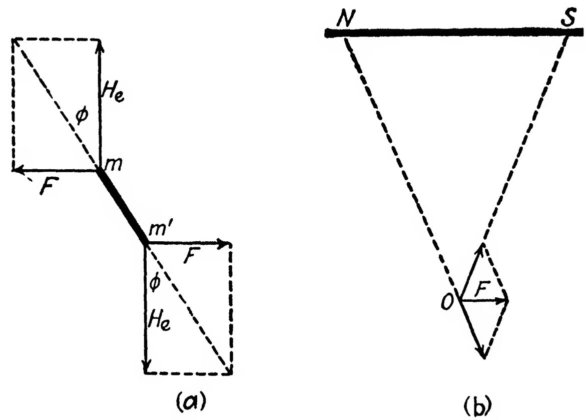


FIG. 57.—(*a*) Deflection of magnetometer needle due to action of the two magnetic forces H and F, acting upon it at right angles to each other. (*b*) The resultant force F at a point O due to a magnet NS (see Fig. 17).

Since the force F, due to the magnetized rod, is normal to H_e at the magnetometer, the relation holds that

$$F = H_e \tan \phi, \tag{104}$$

in which ϕ is the angle which the resultant field makes with the controlling field (see Fig. 57*a*). In any arrangement of the two solenoids, H_e and F will always be the horizontal components of the two forces involved whether the coils are in a horizontal or vertical position. F must now be evaluated in terms of the distance between the poles, the distances between the poles and the magnetometer, and the intensity of magnetization.

Two special cases only will be considered in detail: (1) the horizontal position of the coils shown schematically in Fig. 54

and Fig. 57*b*; (2) the vertical arrangement of the coils indicated in Figs. 56 and 58.

1. *Horizontal Position.*—In Fig. 54 it will be noted that the axes of the coils must be set normal to the controlling field of the magnetometer. In magnetizing the bar, poles N and S will be developed at their respective ends whose forces at M will be, respectively,

$$\frac{N}{\overline{ON}^2} = \frac{\pi a^2 \mathfrak{I}}{\overline{ON}^2} \text{ and } \frac{S}{\overline{OS}^2} = \frac{\pi a^2 \mathfrak{I}}{\overline{OS}^2}. \tag{105}$$

$$(a = \text{radius of rod})$$

The resultant of these two forces will be in a direction parallel to the axes of the coils. From similarity of triangles in Fig. 57*b* it follows that the resultant force is:

$$F : \frac{\pi a^2 \mathfrak{I}}{\overline{ON}^2} = \overline{NS} : \overline{ON} \tag{106}$$

$$F = \frac{\pi a^2 \mathfrak{I} \times \overline{NS}}{\overline{ON}^3}$$

$$= \frac{\pi a^2 \mathfrak{I} L}{\overline{ON}^3}, \tag{107}$$

where L is the distance between the poles. Since

$$F = H_e \tan \phi = \frac{\pi a^2 \mathfrak{I} L}{\overline{ON}^3}, \tag{108}$$

$$\mathfrak{I} = \frac{H_e \times \overline{ON}^3 \times \tan \phi}{\pi a^2 L}$$

$$\mathfrak{I} = K \tan \phi. \tag{109}$$

This equation states that the intensity of magnetization is directly proportional to the tangent of the angle of deflection of the magnetometer needle.

When a *liquid rheostat* is connected in series with the two coils one may steadily vary the field and follow the variations of $\mathfrak{I}$ by the deflection of the magnetometer. The liquid rheostat,[1] used with this method, depends both on change of length and change in cross-section of the liquid conductor for its change in resistance. The fact that the current may be varied without jumps or breaks makes this form of rheostat very useful in magnetic testing.

[1] Williams, *School Sci. and Math.*, **12**, 489, 1912.

2. *Vertical Position.*—Quite frequently it is convenient to have the magnetizing coil in a vertical position. Figures 56 and 58 show an adaptation of the two symmetrical coils to this condition. As in the previous case, N and S are the poles developed in the rod B. L is the distance between the poles, and D_1 and D_2 are the distances from the S and N poles respectively to the magnetometer M. In this arrangement the magnetometer and the two solenoids are placed in an east-west position as the horizontal components of S and N are radial. The horizontal component of the force due to the pole N at M is

$$F_1 = \frac{\pi a^2 \mathcal{I}}{D_2^2}. \tag{110}$$

The horizontal force at M due to the pole S is:

$$F_2 = -\frac{\pi a^2 \mathcal{I}}{D_1^2} \cos SMN = -\frac{\pi a^2 \mathcal{I} D_2}{D_1^2 D_1}. \tag{111}$$

The total horizontal force acting at M is:

$$F = F_1 + F_2 = \frac{\pi a^2 \mathcal{I}}{D_2^2}\left[1 - \left(\frac{D_2}{D_1}\right)^3\right]. \tag{112}$$

Also,

$$F = H_e \tan \phi.$$

Hence,

$$\mathcal{I} = \frac{H_e D_2^2 \tan \phi}{\pi a^2[1 - (D_2/D_1)^3]}, \tag{113}$$

or

$$\mathcal{I} = K \tan \phi.$$

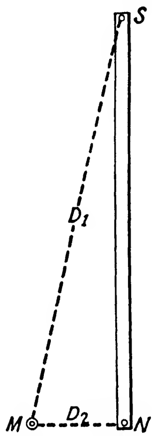


Fig. 58.—Forces acting upon magnetometer in arrangement shown in Fig. 56.

The author[1] has found this arrangement particularly advantageous where he was working with long, slim rods and wished to measure simultaneously the intensity of magnetization and the changes in length produced by a magnetizing force. Figure 59 gives just such a combination in schematic form. C_2 replaces C_2 in Fig. 56. For absolute determinations of $\mathcal{I}$, H_e must be known. Ewing used a tangent galvanometer coil for C_2 and from its constants H_e could be determined.[2]

A disturbance which must be eliminated in this method is the component of the earth's field which is parallel to the axis of the rod. This is usually taken care of by an auxiliary coil wound on the same core as C_1. A direct current through this

[1] Williams, *Jour.* Cleve. Eng. Soc., p. 183, January, 1917.

[2] Starling, "Electricity and Magnetism," p. 272, 1924.

coil can be adjusted to annul just the component of the earth's field which is disturbing the measurements. Having made all the adjustments, the ferromagnetic rod is put into a cyclic state, and then a series of values for ϕ, and the current in the coils is observed as the current is put through a cycle from $+H$ to $-H$ and back to $+H$ again.

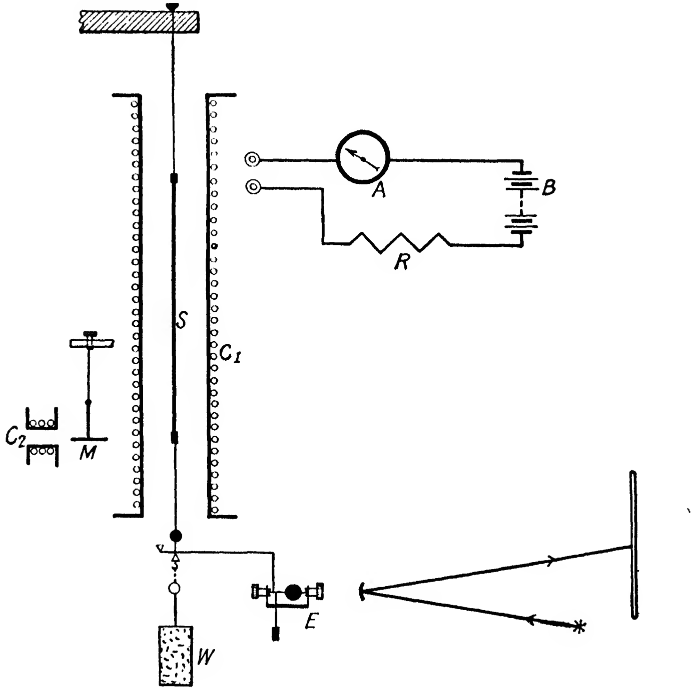


FIG. 59.—Combination outfit for measuring change in length and intensity of magnetization simultaneously.

With the improved forms of magnetometers now obtainable the magnetometric method should be worked over into a more general form of permeameter.[1] In this connection it might not be out of place to say that the *bell-shaped magnets*[2] deserve a wider use in making magnetometers than is now accorded them. Several sizes and forms, both astaticized and unastaticized, are shown in Fig. 60.

The fact that the position of the poles cannot be ascertained with great accuracy makes the magnetometric method less

[1] CIOFFI, *Jour.* Opt. Soc. Amer., **9**, 53, 1924.

[2] WILLIAMS, *Jour.* Opt. Soc. Amer., **16**, 203, 1928.

important for the exact determination of $\mathfrak{I}$. One may get absolute values by the ballistic method, and by means of the standard thus developed use the magnetometric method for comparative tests. In this procedure an attempt would be made to keep a, L, and distances from poles to magnetometer constant for all specimens. For comparative tests the magnetometric method is in many ways ideal.

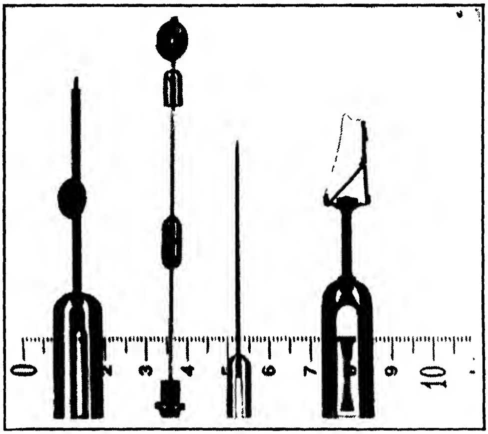


FIG. 60.—Various sizes and combinations of the bell-shaped magnet.

b. Ballistic Method.—In this method the total flux B is measured. In the magnetometric method $\mathfrak{I}$ was measured. In the latter method the magnetometer was the indicating instrument. In the ballistic method, a coil and a ballistic galvanometer, the equivalent of a magnetometer, will be the instruments of measurement. If the coil connected to the galvanometer has a magnetizing force surging through it, normal to its turns, an electromotive force will be developed in it which will give rise to a deflection in the galvanometer. This deflection is proportional to the magnetizing force applied,

$$H = kd. \tag{114}$$

The deflection will be quite different, however, if a bar of ferromagnetic material is in the coil when the magnetizing force is applied. The new deflection will be expressed by the equation,

$$B = kD, \tag{115}$$

in which k is the same constant as in (114). Unless the exploring coil lies close to the surface of the rod, the flux observed will not be true flux because some of the lines of force of the applied field go between the coil and the rod. This correction takes the form,

$$B_{\text{true}} = B_{\text{observed}} - \frac{A - a}{a} H, \tag{116}$$

where a is the cross-section of the rod and A is the inner area of the coil.

If a solenoid is used for producing the magnetizing force,

$$H = 4\pi n I = kd$$
$$B = kD.$$

Therefore,

$$B = \frac{4\pi n I D}{d}. \tag{117}$$

In using such a simple outfit, it is desirable to have the specimens being studied in the form of long, slim rods. As is evident from (117) one may take a series of readings for B and H and so obtain a normal induction curve.

Since magnetic lines of force are closed ones, the nearer the magnetic circuit approaches this same condition the less trouble will there be by corrections for magnetic leakage, for demagnetizing factors, and for kindred disturbances. Rowland[1] made use of the toroidal electromagnet for getting the normal induction curve of its core. One objection to this form of the material for magnetic testing is that each ring has to have its own primary and secondary coils. This is a bothersome procedure where there is a large number of samples for routine testing. Academically, it is a splendid outfit and will be discussed here because it gives an excellent approach to the more modern methods now in use.

Before proceeding to the theory of the *ring-ballistic method* it will be worth while to recall certain fundamentals regarding the quantity of electricity in an inductive circuit. In Eq. (1) it was shown that the number of lines of force, cut by a conductor in a unit of time, gave a measure of the electromotive force developed. This was expressed in the form

$$\text{Emf} = \frac{NEv}{10^8} \text{ volts,} \tag{1}$$

[1] ROWLAND, *Philos. Mag.*, **46**, 140, 1878.

viz., that the electromotive force is equal to the rate of change of the number of lines of force threading through the loop. If there had been more than one loop the above would have been multiplied by the number of loops linked with the number of lines of force. If ϕ is the total number of lines of force threading through a coil of T turns, then the product ϕT is called the *flux turns* of the coil. Equation (1) may be written:

$$\text{Emf} = \frac{-d(\phi T)}{dt} \times 10^{-8} \text{ volts.} \tag{118}$$

If A is the effective area of the coil, $\phi = \mu HA$ and, therefore,

$$\text{Emf} = \frac{-d(\mu HAT)}{dt} \times 10^{-8} \text{ volts.} \tag{119}$$

The negative sign is written in here to indicate "that the current induced is directed so that its magnetic field reacts with the field H to oppose the motion, as required by the law of conservation of energy."

When, by one of several means, the flux turns of a coil are made to vary suddenly, a transient current will be set up in the circuit, which is, however, variable and difficult to measure. The quantity of electricity which passes through the circuit can be, nevertheless, readily determined. Ohm's law applies to these transient currents $I = E/R$. Recalling that emf $= d(\phi T)/dt$ and $I = dQ/dt$, Ohm's law may be written.

$$dQ = \frac{d(\phi T)}{R}. \tag{120}$$

Integrating this expression between the initial and final values of the flux turns we have:

$$Q = \frac{1}{R}\int_{\phi_2 T}^{\phi_1 T} d(\phi T) = \frac{\phi_1 T - \phi_2 T}{R};$$

whence,

$$Q = \frac{\Delta(\phi T)}{R}. \tag{121}$$

If the flux turns are changed, by ϕ's being suddenly reversed, then $\Delta(\phi T) = 2\phi T$ because the lines of force are first withdrawn and then immediately reestablished in the opposite direction. Here

$$\phi_1 = \phi_2 = \phi, \text{ and } Q = \frac{(\phi_1 - (-\phi_2 T)}{R} = \frac{2\phi T}{R}. \tag{122}$$

Applying these equations to the coil connected to the ballistic galvanometer it is evident that the quantity of electricity sent through the galvanometer circuit will be equal to the change in the flux turns of the coil divided by the total resistance of the circuit. Knowing the flux turns of the coil and the total resistance we are in a position to determine Q. This is a principle that will be used over and over again in magnetic measurements and should be grasped once for all. It is just this equation which is employed in working with an earth inductor (see Sec. 67).

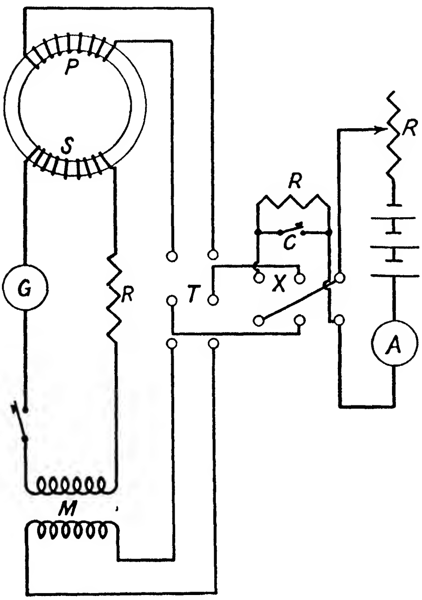


FIG. 61.—Diagram of connections for ballistic method in studying magnetic properties of rings.

There still remains the problem of finding B for an inductive circuit containing a ferromagnetic body as is shown in Fig. 61. If the galvanometer in this circuit is calibrated so that D in (117) is known in terms of the induced flux, then B can be determined. This is just the purpose of the mutual inductance M in Fig. 61. A reversal of a known flux in M gives a deflection d by which the unknown deflection D of (117) may be evaluated. The expression for the relation between the mutual inductance and the deflection of the galvanometer is analogous to that of (117).

Suppose two circuits are laid side by side as in Fig. 61. If a current I flows in the primary circuit of M there will be a certain field developed in the loop which will be proportional to I. This field will not be uniformly distributed, but for any point within the loop, there the field will vary as I. The field produced by the current I will, according to Faraday's discovery, induce an electromotive force in the secondary loop of M. It follows, therefore, that the flux turns of the secondary will also be proportional to I, or

$$\phi T = MI. \tag{123}$$

M is a geometric constant independent of the value of I. M is called the mutual inductance of the two circuits.

$$M = \frac{\phi T}{I}. \tag{124}$$

There are several ways in which M may be defined.[1] For present purposes Eq. (124) will suffice. Expressed in words, this equation defines the mutual inductance of two circuits as the ratio of the flux turns to the current producing the primary field. A pair of circuits has a mutual inductance of one henry when an electromotive force of one volt is induced in one of them by a change of one ampere per second in the other.

It has just been shown that

$$Q = \frac{\phi_1 T - \phi_2 T}{R}.$$

From (123) it follows that

$$Q = \frac{MI_1 - MI_2}{R}. \tag{125}$$

If the initial current in M is broken and reversed the flux turns decrease from $\phi_1 T$ to 0 and then increase from 0 to $\phi_2 T$. Hence,

$$Q = \frac{M[I_1 - (-I_2)]}{R} = \frac{2MI}{R}, \tag{126}$$

since $I_1 = I_2 = I$.

By the use of standard current inductors whose M can be accurately calculated, Eq. (126) affords a means for calibrating a ballistic galvanometer. The intent of the past few pages is to show that in getting a BH curve, the only way this can be done ballistically is to calibrate a ballistic galvanometer to give deflections in terms of flux.

To sum up the theory of the ring-ballistic method we may proceed as follows: the galvanometer circuit in Fig. 61 is arranged with a constant total resistance R_t, which can be set to any desired value by the rheostat R. Either the flux in M or in S may be reversed and each gives its deflection independent of the other. For the mutual inductance in Fig. 61,

$$Q_1 = \frac{2MI}{R_t} = kd, \tag{127}$$

while for the coil S,

$$Q_2 = \frac{2BAT}{R_t} = kD. \tag{128}$$

[1] SMITH, "Electrical and Magnetic Measurements," p. 183, 1917.

Dividing (128) by (127),

$$\frac{BAT}{MI} = \frac{D}{d}$$

$$B = \frac{MID}{ATd},$$

or, if one makes $d = D$,

$$B = \frac{MI}{AT}$$

$$B = \frac{MI}{AT} \times 10^8 \text{ gausses,} \tag{129}$$

if M is expressed in henrys and I in amperes.

Couple this last equation with (54) and all the quantities necessary for finding a normal induction curve or a hysteresis curve have been determined.

The late eighties and early nineties of the nineteenth century saw unusual activity in studying the magnetic properties of ferromagnetic bodies. It was early recognized that the desirable features of the ring method should be conserved, particularly the closed magnetic circuit. The separate primary and secondary coils for each ring, however, was a nuisance. This led Hopkinson,[1] Ewing,[2] and others[3] to devise a *yoke system* which allowed putting in and taking out the specimen in rod form without disturbing the windings. Hopkinson had a massive yoke something like that shown in Fig. 62. Symmetrically placed in this yoke was a ferromagnetic rod B, cut so that one-half of it could be quickly withdrawn. The two ends of the rods abutting on each other were carefully ground so that as good magnetic contacts as possible would be formed. Over each half of the rod was placed a magnetizing coil with a small exploring coil between them. While the rod was magnetized, one-half of the rod B was

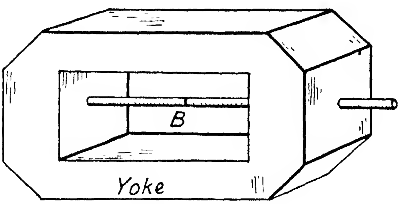


FIG. 62.—Yoke in which is thrust a bar B. If a coil surrounds the rod, a complete magnetic circuit is made through the yoke. It offers some of the advantages of the ring specimens.

[1] HOPKINSON, "Original Papers," vol. II, p. 154; *Proc. Roy. Soc.*, **176**, 455, 1885.

[2] EWING, "Magnetic Induction in Iron and Other Metals," 3d ed., p. 362.

[3] ILIOVICI, *Bull.* Soc. int. des élec., 3d ser., **3**, 581, 1913.

quickly removed allowing the exploring coil to be jerked by a rubber strip to a point where it was out of the field of the rod. The deflection of the galvanometer attached to the exploring coil was a measure of the magnetic flux in the rod. The great difficulty with this method was to determine without too serious an error the magnetomotive force applied to the rod. Also considerable care had to be paid to the air gaps between the

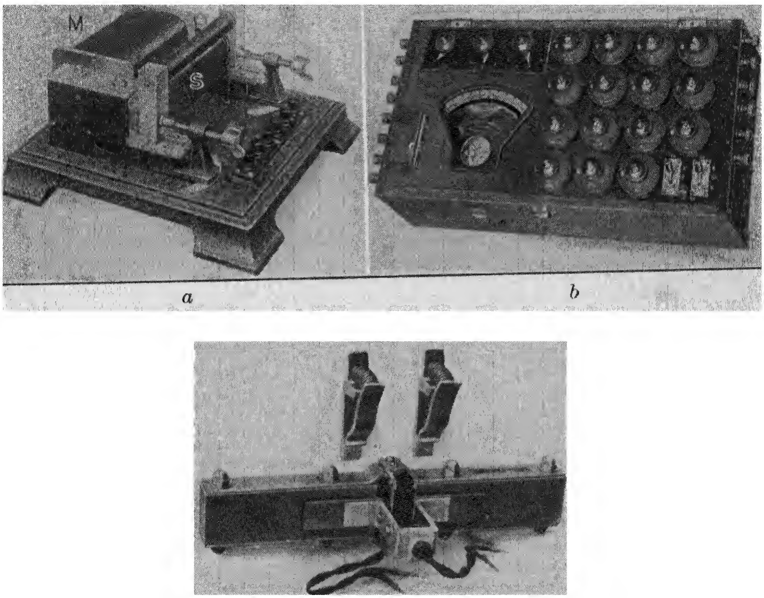


FIG. 63.—(*a*) Fahy's *Simplex permeameter*. (*b*) Control box for Fahy's permeameter. (*c*) Adjunct to the Fahy permeameter for testing materials in strong fields.

ends of the rod and contacts with the yoke. Spooner[1] gives a review of a number of different kinds of permeameters.

From the more modern methods, just one will be selected for discussion in this chapter. This is Fahy's[2] *Simplex permeameter*, shown in Fig. 63*a*. It involves the principle of Chattock's[3] *magnetic potentiometer* which other permeameters of this type

[1] SPOONER, "Properties and Testing of Magnetic Materials," p. 204, 1927.
[2] FAHY, *Sci. Papers* Bur. Stand., **306**, 267, 1917;
SANFORD, *Jour. Research* Bur. Stand., **4**, 703, 1930.
[3] CHATTOCK, *Philos. Mag.*, **24**, 94, 1887.

use, and at the same time is a very convenient outfit. Figure 64 shows a schematic drawing of the outfit. M is a U-shaped electromagnet across the poles of which the specimen is clamped. The core of the electromagnet and the specimen form a closed magnetic circuit. The coil on M is the primary, and the coil S surrounding the specimen is the secondary. The second secondary coil S' acts as the potentiometer. In Fahy's permeameter the primary circuit has no form whereby the magnetomotive

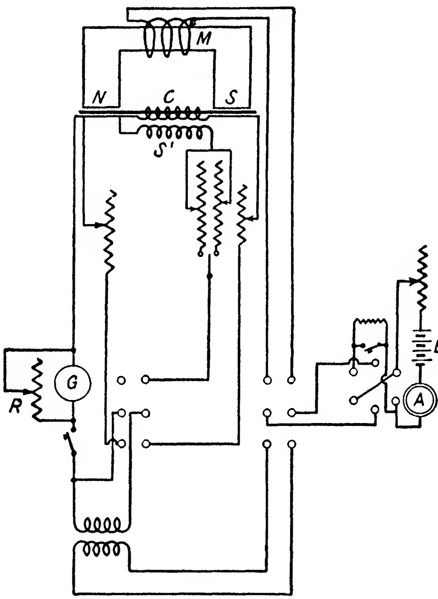


FIG. 64.—Connections between control box and permeameter.

force applied to the specimen can be calculated as was done in the ring form. In the first place the magnetomotive force of the primary coil is not that applied to the specimen, and secondly the core of the primary is not a part of the specimen. The magnetomotive force applied to the rod must be determined experimentally and this is done by means of Chattock's potentiometer principle.

It will help in the understanding of Fahy's permeameter to think of the two secondary coils C and S' (Fig. 64) as two magnetic circuits in parallel with the same magnetomotive force applied to them just as two electric circuits have the same elec-

tromotive force if they are connected in parallel. Figure 65 will assist in visualizing this point of view. The ends of the specimen B are inserted in cavities within the poles of the heavy yoke and thus form a continuous magnetic circuit. N and S are large iron end-plates. When a magnetic field is developed by the coil wound on the yoke, it produces between the plates N and S the same magnetomotive force for the coil as for the bar. If the magnetomotive force along the coil S' (Fig. 64) can be found then the magnetomotive force along the specimen will be known also. It will be noted in Fig. 64 that M, G, S, S', R, and B are connected as the corresponding parts were in Fig. 61. In exactly the same way as for the ring method, the magnetomotive force for the coils C and S' may be determined. In the case of the coil S',

$$H = \frac{2MI}{AT} \times 10^8 \text{ gausses,} \tag{130}$$

while for the coil C,

$$B = \frac{2MI}{A'T'} \times 10^8 \text{ gausses.} \tag{131}$$

Thus for the Fahy permeameter the necessary units are furnished for running a normal induction curve.

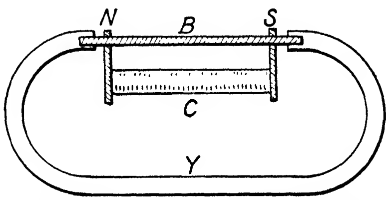


FIG. 65.—A magnetic field is formed by the yoke Y between the magnetic contact shoes N and S. Since the magnetomotive force between N and S is the same for both B and C, the flux through B and C will be inversely as their reluctances.

Figures 63a, b, and c show the form of Fahy's permeameter as put out commercially. The coils M, S, and S' are clearly seen in Fig. 63a. These coils are connected to the "control box" shown in Fig. 63b. In the control box are conveniently and compactly arranged the switches, ammeter, resistances, and mutual inductance shown in Fig. 64. The ballistic galvanometer is connected to the control box by long, twisted lead wires which will place the instrument some distance from the powerful electromagnet M.

Fahy has also designed a high magnetization adapter Fig. 63c, as an adjunct to the Simplex permeameter. The purpose of this adapter is to make possible the testing of materials under magnetizing forces as high as 1,000 to 1,200 gilberts per centimeter. This is accomplished by concentrating the total magnetomotive force of the yoke over a short length of the test specimen without

additional battery voltage. Using the description of the designer:

> This device is also of value in permitting the test of very short specimens in the lower range of magnetizing forces.
>
> The adapter consists of two laminated sections, spaced by a gap, which are placed so as to span the polar faces of the Simplex permeameter yoke, the H and B coils of the latter being removable for this purpose. The test sample is placed across the gap and within a special B coil located at that region.
>
> An aluminum arbor, which carries thin extension blocks and a special H coil, is placed across the face of the sample. Both arbor and sample are secured by means of the clamps shown in the illustration. The adapter H and B coil leads connect to binding posts on the Simplex permeameter base.
>
> The high-magnetization adapter is designed for the testing of bar and laminated specimens, 2½ in. or over in length, and of section up to 1¾ in. by ⅝ in.

Specific details for using this form of permeameter accompany the instrument.

c. Hysteresis Loss by Fahy's Permeameter.—The fixed-point method is the procedure whereby hysteresis data are obtained by means of the Fahy permeameter. Hysteresis determinations are made from chosen values of induction. The hysteresis loss for any given material depends upon the value of the maximum induction selected. For points beyond that of saturation, the hysteresis loss approaches a constant value.

In Fig. 48 any value of H, say $H_{\max}$ may be chosen and the value of B, corresponding, will then be $B_{\max}$. These two values of B and H will be definitely determined just as for the normal induction curve. If $H_{\max}$ is suddenly decreased to H_e, $B_{\max}$ will also be decreased and take a value B_q.

$$B_{\max} - B_q = dB, \tag{132}$$

and this variation in flux will give rise to a deflection of the galvanometer from the coil S, such that

$$dB = \frac{2MID}{ATd}, \tag{133}$$

where the right-hand member of the equation will be interpreted in terms of the deflection of the galvanometer or in terms of flux. Therefore,

$$B_q = B_{\max} - dB. \tag{134}$$

Similarly H_e will be determined by changing the flux through S', such that

$$H_e = H_{\max} - dH, \tag{135}$$

where dH will be determined in terms of flux also.

Since $B_{\max}$ and $H_{\max}$ were definitely located, the above procedure gives us the fixed point B_q, H_e on the hysteresis curve. The specimen is once more taken to the same condition $B_{\max}$ and $H_{\max}$ as previously determined. The resistance R' is made larger so that when it is thrown in series with the magnetizing current, $H_{\max}$ will drop to H_d, and correspondingly $B_{\max}$ will drop to B_p. This, in turn, will give a new fixed point B_p, H_d. This process, continued for other fields, will give any magnetization process desired.

33. Para- and Diamagnetic Susceptibilities.—In the development of our modern electric light and power equipment it was natural, from the historical point of view, that ferromagnetic bodies should have more attention paid to them than the para- and diamagnetic substances did. Also, the magnetic effects in ferromagnetic bodies were much larger than in the others and so, from the laboratory point of view, could be studied more easily than para- and diamagnetic bodies.

From the standpoint of the development of a comprehensive theory of magnetism, however, this emphasis on a study of ferromagnetics was not warranted. It may be safely asserted that the greatest advances, along modern theoretical lines, have been supported, in the main, by the experimental work which has been done on para- and diamagnetic bodies.

In a study of the phenomena of magneto-magnetics, therefore, a much fuller consideration of para- and diamagnetic substances is demanded than was given in Sec. **4.** These substances show only a feeble reaction to a magnetic field, and yet the improved technique of recent years has enabled many investigators to study these substances with most valuable results. Their value seems to lie in the fact that the results have to do, in large part, with atomic phenomena since they are expressed so often in terms of atomic susceptibilities. As a suggestion it might be said that, if ferromagnetic bodies are to be studied with a view to their contributing to magnetic theories, they must be studied in the form of single crystals. This is a field which

is beginning to receive considerable attention[1] and needs a great deal more.

Faraday seems to have been the first to recognize that all bodies are affected in one way or another by a magnetic field. The early investigators confined their observations largely to noting whether a body was attracted toward or repelled from the poles of a magnet. Gradually these observations took on a quantitative aspect in that relative values were obtained for these forces of attraction and repulsion. The first outstanding research in this field is the classical work of Curie.[2] His work, probably, did more to open up this field and stimulate research in it than any other work up to the present time. It is an exceedingly important field and one which needs further development. Special attention should be paid to the purity of the substances used for observations. This point cannot be too strongly emphasized.

In order to understand the methods for studying the magnetic properties of para- and diamagnetic bodies it will be necessary to digress for a moment and consider some of the fundamental principles involved.

34. Nature and Magnitude of the Forces Acting on Bodies in a Magnetic Field.—We have seen that when a bar of iron or other ferromagnetic substance is brought under the influence of a magnetic field it becomes magnetized by induction. The magnetic poles thus induced react on the magnetizing field and the forces involved may be observed and calculated. This is true not only for ferromagnetic but also for para- and diamagnetic substances, even though in the latter the permeability is very small as compared with ferromagnetic bodies.

In studying the behavior of para- and diamagnetic bodies in a magnetic field it will clarify one's ideas to picture the condition of the lines of magnetic force which permeate certain combinations of media.

a. Field Parallel to Surface of Separation.—In Fig. 66 let N and S represent rather extensive pole pieces of an electromagnet so that between them there is a region in which a uni-

[1] Weiss, "Thèses prés à la faculté des sci. de Paris," 1900;
Webster, *Proc. Roy. Soc.*, **107**, 496, 1925; **109**, 570, 1925;
Honda and Kaya, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 721, 1926;
Honda and Mashiyama, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 755, 1926;
Gerlach, *Ztsch. für Phys.*, **38**, 828, 1926.

[2] Curie, "Thèses prés à la faculté des sci. de Paris," 1895.

form field may be obtained. Into the space between the poles two pieces of different materials may be fitted so that each one will occupy one-half of the space. One medium will have a permeability of μ_1 and the other μ_2. For the present discussion, μ_1 and μ_2 are not widely different and not much larger than unity. Inasmuch as the magnetomotive force between N and S is uniform all over the faces of the pole pieces, the intensity of field in the two media must also be the same, *viz.*, $H_1 = H_2$. On the other hand, the flux will be such that $\mu_1 H_1 \neq \mu_2 H_2$, because $\mu_1 \neq \mu_2$. The flux in Medium 2 is μ_2/μ_1 greater than in Medium 1 if $\mu_2 > \mu_1$. This is indicated in Fig. 66 by the number of lines drawn across Medium 1 and Medium 2 respectively.

b. Field Normal to Surface of Separation.—In Fig. 67 is shown a second combination of media. The two equal-sized blocks

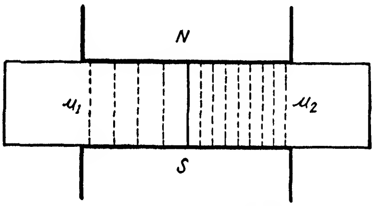


FIG. 66.—Two paramagnetic substances placed between the poles of a magnet. Surface dividing the two bodies parallel to the field.

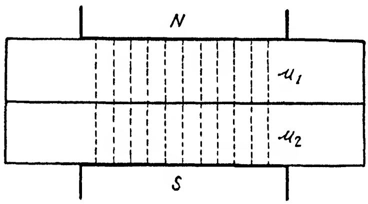


FIG. 67.—Two paramagnetic substances placed between the poles of a magnet. Surface dividing the two bodies normal to the field.

of material with permeabilities μ_1 and μ_2 are placed between the poles of the electromagnet. The surface of separation is normal to the magnetic lines of force. The magnetomotive force between the poles produces a definite number of lines of flux across the two media. In this case there is no discontinuity in flux in crossing the boundary surface. For this combination of media $\mu_1 H_1 = \mu_2 H_2$, and therefore $H_1 \neq H_2$, since $\mu_1 \neq \mu_2$. The field intensity thus seems to make a jump at the boundary surface, as though there were a surface layer of magnetic poles there to augment or decrease the field depending upon whether $\mu_1 \gtrless \mu_2$. This fictitious layer of magnetism may be thought of as having a surface density of m and so $4\pi m$ lines of induction radiate from the surface per unit area. The jump at the boundary will be $-4\pi m = H_1 - H_2$ or $4\pi m = H_2 - H_1$. The sign will depend upon whether $\mu_1 \gtrless \mu_2$. In the two media the relation holds that $H_1/H_2 = \mu_2/\mu_1$ or $H_2 = \mu_1/\mu_2 H_1$,

whence,

$$-4\pi m = H_1 - \frac{\mu_1}{\mu_2}H_1 = H_1\left(\frac{\mu_2 - \mu_1}{\mu_2}\right) \tag{136}$$

or,

$$m = \frac{H_1}{4\pi}\left(\frac{\mu_1 - \mu_2}{\mu_2}\right). \tag{137}$$

The sign of m will also depend upon whether H_1 is directed away or toward the medium of high permeability at the bounding surface.

Suppose a positive magnetic pole P could be isolated and surrounded by a limitless medium whose permeability is μ_1. The

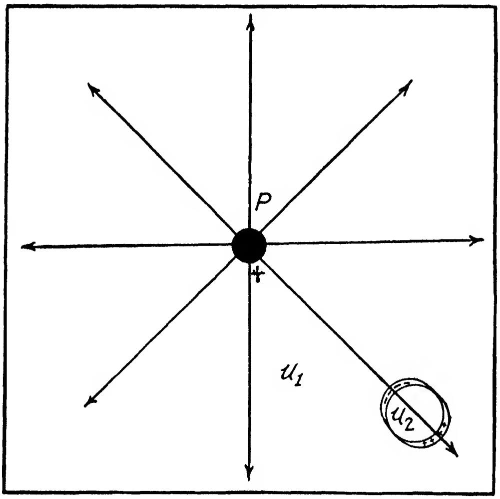


FIG. 68.—The magnetic field of an isolated pole P immersed in a medium of permeability μ_1.

magnetic lines of force will go out radially from the pole to infinity and the intensity will vary according to the inverse square law (see Fig. 68). Let another body of permeability μ_2 be introduced into this field of P. The lines of force will pass from the mediun μ_1 into medium μ_2. According to Case (*b*), which has just been discussed, this will produce a fictive layer of magnetic poles on the boundary surface wherever there is a component of the field normal to the surface.

If $\mu_2 > \mu_1$ then m is negative and the medium μ_2 will be attracted toward the positive pole P. If $\mu_2 < \mu_1$ then just the reverse will happen and the medium μ_2 will move away

from P. This will indicate why para- and diamagnetic substances behave as they do in a non-uniform magnetic field. It will visualize the reason why a diamagnetic body moves in a non-uniform field from points of high to low intensity, and also why paramagnetic bodies behave otherwise. This point of view is particularly obvious if the body with permeability μ_2 is a sphere of soft iron. Using the specialized expressions for ferromagnetism we would say that on the side of the sphere nearest the pole P a negative pole m is induced which is attracted by the positive pole producing the field.

In the two cases shown in Figs. 66 and 67, the simplest conditions have been taken, *viz.*, when the imposed magnetic field was (*a*) parallel to and (*b*) normal to the boundary surface between the two media. A more general case to be considered is when the boundary surface forms an acute angle with the direction of the field. If the surface is thus struck by the magnetic lines of force, refraction occurs and the law whereby this ensues will be deduced.

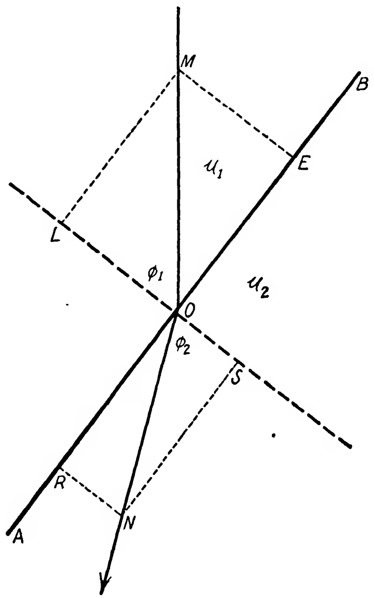


Fig. 69.—The refraction of magnetic lines of force at the boundary of two media.

Suppose AB (Fig. 69) is the cross-section of the surface of separation between two media whose permeabilities are μ_1 and μ_2 respectively. MO is the direction of the magnetic field in the first medium, and ON is that in the second. Indicative of what is occurring all over the surface, these two directions may be resolved into components parallel and normal to the dividing surface. EO is the component of MO parallel to the surface of separation. OR is the corresponding component for ON in the second medium. From considerations of Fig. 66 it has been shown that the components parallel to the boundary surface remain constant, or

$$EO = OR. \tag{138}$$

The components normal to the surface suffer a discontinuity. This was shown in Fig. 67. The relation between these two components is expressed by the proportion:

$$LO:OS = \mu_2:\mu_1, \tag{139}$$

i.e., the normal components are inversely proportional to their respective permeabilities. The components OR and OS give a resultant which is different in magnitude and direction from that of the original field MO. Refraction of the magnetic lines of force have therefore occurred, and the law of refraction may be deduced from Fig. 69. If ϕ_1 and ϕ_2 are the incident and refracted angles, then,

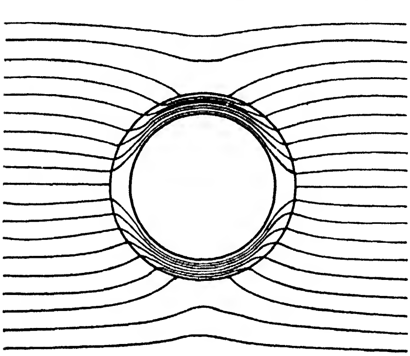


FIG. 70.—Any object placed inside of the hollow cylinder will be shielded from outside magnetic forces.

$$\tan\phi_1 = \frac{LM}{LO} \text{ and } \tan\phi_2 = \frac{NS}{OS}, \tag{140}$$

but

$$LM = NS \text{ and } LO:OS = \mu_2:\mu_1.$$

Therefore,

$$\tan\phi_1:\tan\phi_2 = \mu_1:\mu_2 \tag{141}$$

or,

$$\frac{\tan\phi_1}{\tan\phi_2} = \text{constant}. \tag{142}$$

In the case that μ_2 is for iron and has a very high value, and μ_1 is for air, it follows that even for very small departures from normal incidence very strong refraction of the lines of force occurs.

It is due to this tendency of the magnetic lines of induction to be strongly refracted when they pass at an oblique angle into a second medium of high permeability, that the high *shielding effect of iron* is obtained. This shielding power of iron is used in many ways to protect sensitive instruments from outside magnetic forces. The armored galvanometer with its moving magnet system secures steadiness of deflection by including magnets and coils within several concentric iron shells which may be spherical or cylindrical.[1] Figures 70 and 71 indicate by means of the lines of force how these shells produce their shielding power. Engineers in electric light and power plants use steel

[1] DUBOIS, *Electrician*, **40**, 317, 1898; WILLS, *Phys. Rev.*, **24**, 243, 1907; NICHOLS and WILLIAMS, *Phys. Rev.*, **27**, 250, 1908.

cases about their watches. How effective they are is a question. For shielding from fields, like the earth's field, the new alloy, "permalloy," seems to offer some suggestions. Complete magnetic shielding is not possible as is the case with electric shielding.

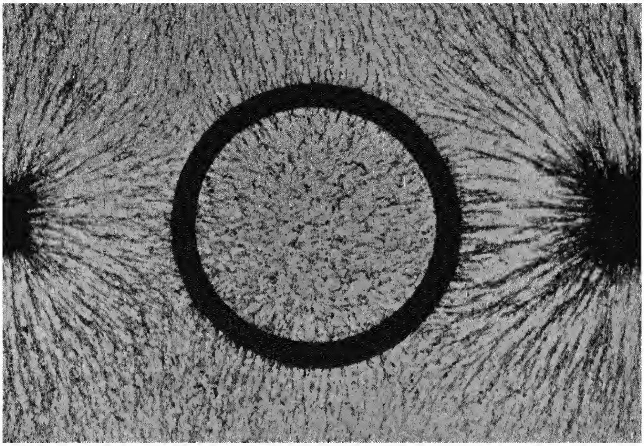


FIG. 71.—The iron filings inside the ring are not lined up in any particular direction. Fair shielding is thus secured, but it is not complete.

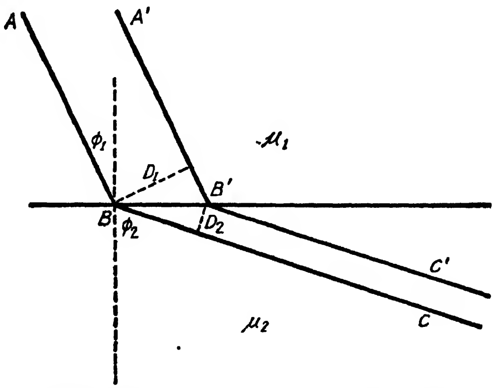


FIG. 72.—Magnetic induction at the boundary of two media.

One factor seems to be that perfectly homogeneous shells are not obtainable, and as a result induced polarity occurs in the shields themselves, which may give rise to lines of force in the space where it is desired to eliminate the magnetic fields.

This refraction of the oblique lines of induction at a boundary surface causes an increase in the lines representing the flux density. From the geometrical relations, the relative values of the flux densities and field intensities in the two media may be calculated and shown to agree with the conditions which have just been discussed.

In Fig. 72 ABC and $A'B'C'$ represent the cross-section of a bundle of lines of induction in the two media. The perpendicular distances between these lines in the two media are D_1 and D_2 respectively.

$$D_1:D_2 = \mu_2 H_2:\mu_1 H_1, \tag{143}$$

where $\mu_1 H_1$ and $\mu_2 H_2$ are the lines of flux per square centimeter.

Furthermore,

$$D_1 = BB' \cos \phi_1, \tag{144}$$

and

$$D_2 = BB' \cos \phi_2,$$

or

$$D_1:D_2 = \cos \phi_1:\cos \phi_2 = \mu_2 H_2:\mu_1 H_1. \tag{145}$$

This equation says that the magnetic flux per square centimeter in the two media varies inversely as the cosines of the incident and refracted angles.

From Eqs. (141) and (145) it follows that

$$H_1:H_2 = \sin \phi_2:\sin \phi_1. \tag{146}$$

This equation says that the field intensities in the two media vary inversely as the sines of the incident and refracted angles.

1. For the case that $\phi_1 = 90°$, $\sin \phi_1 = 1 = \sin \phi_2$. *For the components of flux parallel to the boundary surface,*

$$\boxed{H_1 = H_2} \tag{147}$$

2. For the case that $\phi_1 = 0°$, $\cos \phi_1 = 1 = \cos \phi_2$. *For the components of flux normal to the boundary surface,*

$$\boxed{B_1 = B_2} \tag{148}$$

These two special cases derived from general conditions are precisely the two special cases with which the discussion of this section was started.

In the preceding paragraphs it has been shown that any substance whatsoever, when introduced into a magnetic field, becomes in effect a magnet, either by induction or by the fictive poles developed on the surface. Qualitatively, it could be shown why paramagnetic bodies were attracted toward the magnetic pole P in Fig. 69 and why diamagnetic bodies were repelled. It is now desired to have an expression for these forces that will lead to quantitative results.

It is primarily a problem of determining what are the forces acting upon a magnet when placed in a magnetic field. Section **8** showed that there would be a couple acting upon the mag-

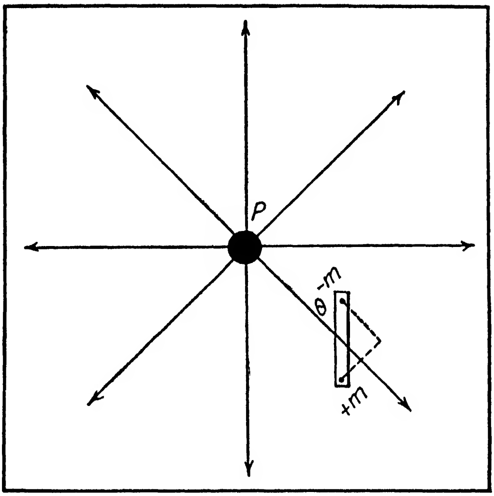


FIG. 73.—Action of the magnetic field of an isolated pole P on the field of a bar magnet.

net tending to turn the axis parallel to the field. The conclusion to be drawn from the preceding paragraphs is that if the field is uniform, no other process operates on the magnet. If, on the other hand, the field is not uniform then two processes are present, one to produce rotation, and the other translation.

Figure 73 illustrates these two processes and is a special case of the conditions shown in Fig. 68. P is a positive magnetic pole with its lines of force radiating out to infinity. Into this non-uniform magnetic field is brought a small *permanent* magnet of length l and pole strength m. It is at once evident that the magnet will endeavor to turn its axis parallel to the lines of force and, inasmuch as $-m$ is nearer P than $+m$, the magnet as a

whole will tend to be translated toward P. The couple tending to produce rotation was given by Eq. (6),

$$C = MH \sin \Theta, \tag{6}$$

where Θ is the angle the axis of the small magnet makes with the field H, and $M = ml$, the magnetic moment of the same magnet.

The force producing translation may be deduced from a consideration of the potential energy of the small magnet with respect to the field. Let the magnet be brought from infinity to the point it occupies in Fig. 73. The work done in bringing up the positive pole is just equal to the work done by the negative in coming up to the point occupied by the positive pole. The extra work performed by $-m$ when the magnet maintains the angle of Θ with the field is the work of carrying $-m$ along the line of force, a distance equal to the component of the distance between the poles parallel to the field, *i.e.*,

$$W = (Hm)(l \cos \Theta).$$

This is the potential of the magnet and may be written:

$$W = MH \cos \Theta. \tag{149}$$

The variation of W along any direction gives us the force operating along that same direction. Let x be the direction along the line of force passing through the magnet. The force acting on the magnet to move it will be:

$$f = \frac{dW}{dx} = M\frac{dH}{dx} \cos \Theta. \tag{150}$$

If the magnet is free to move in space, $\Theta = 0°$, and

$$f = M\frac{dH}{dx}. \tag{151}$$

Θ need not have appeared at all if the magnet had been brought from infinity with the positive pole following directly in the track of the negative pole. In fact, Θ will not appear experimentally if the body is a magnet by induction, since the poles will always be induced in the body so that the axis joining them will always be parallel to the field. In the discussion of Fig. 73 a small permanent magnet was considered, so that M would be constant. If, for instance, a piece of soft iron is used, the pole strength m will be a function of the field strength

and thus M will vary from point to point. In experimental work care is always taken that the body be placed at a definitely known position in the magnetic field, for which the body will have a definitely known value of M and Eq. (151) will hold.

For a non-uniform magnetic field, therefore, the action on a body, magnetically polarized by the field, consists of a couple,

$$C = MH \sin \Theta, \tag{6}$$

and a force,

$$f = M\frac{dH}{dx} \cos \Theta, \tag{150}$$

wherein the couple is proportional to the field strength H, and the force of translation is proportional to the gradient of the field.

Considerable space has been allotted to the discussion of the behavior of a body in a magnetic field when it is free to move. This emphasis has been permitted because it seemed fundamental in understanding correctly the experimental procedure which is presently to be reviewed.

35–36. Measurement of Susceptibilities. *a. General Theory.*—Equation (151) becomes a very important one because it may be tested experimentally. Recalling that $\mathcal{I} = M/v$ and $\chi = \mathcal{I}/H\rho$, Eq. (151) may be written in the form,

$$f = \chi m H\frac{dH}{dx}, \tag{152}$$

where χ is the specific susceptibility of the mass m, freely suspended in a magnetic field H. The specific susceptibility is the quantity in which we are most interested, therefore,

$$\chi = \frac{f}{mH\dfrac{dH}{dx}}. \tag{153}$$

All of the members of the right-hand side of the equation may be determined experimentally. Values of H and dH/dx are usually found by exploring coils. However, one may determine χ for an unknown substance without knowing H and dH/dx, if the same measurements have been made with a known test-specimen.[1]

b. Faraday's Method.—Faraday[2] was the first to show the "universality" of magnetism and did a prodigious amount of

[1] Ishiwara, *Sci. Repts. Tôhoku Imp. Univ.*, **9**, 236, 1920.

[2] Faraday, "Experimental Researches," vol. III, p. 497.

work in studying and classifying substances of low susceptibility into two classes, dia- and paramagnetic. His method of giving freedom to the body in the magnetic field was to suspend it from one arm of a torsion balance as shown in Fig. 74. The non-uniform field was obtained by pointed poles as shown in Fig. 75, where C is the cross-section of the mass suspended in the magnetic

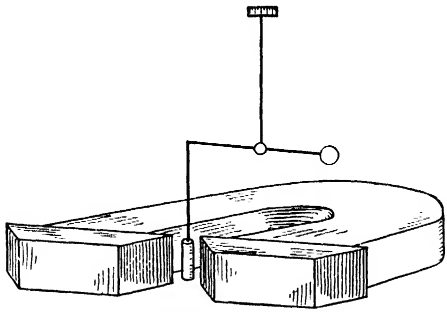


FIG. 74.—Faraday s magnetic balance for studying susceptibilities.

field at about 1.2 cm. from the point of greatest intensity. Faraday did not apply Eq. (152) to the absolute determination of $\mathcal{X}$, but distinguished between diamagnetic and paramagnetic bodies and obtained relative values in terms of water in air as a standard. As has been pointed out diamagnetic bodies will move away from the strongest part of the field and paramagnetics will go in the opposite direction. The translatory force acting upon

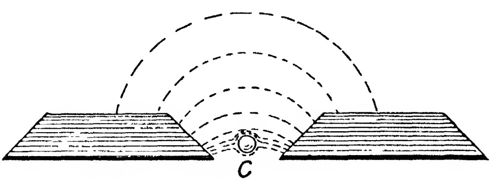


FIG. 75.—Cross-section of magnetic field and specimen C in Faraday's magnetic balance.

the specimen was measured by the turn of the torsion head T (Fig. 74), necessary to bring the sample back to zero position. The specimen was cylindrical in form for solids while gases and liquids were contained in a bulb.

A cylinder of glass was suspended first in air, and then in water. If K_g, K_a, and K_w are the susceptibilities of the glass, air, and water respectively, then from the angular readings on

the torsion head the susceptibility of any substance might be determined in terms of this standard:

$$K_g - K_a = c\Theta_a$$
$$K_g - K_w = c\Theta_w$$
$$K_w - K_a = c(\Theta_a - \Theta_w) = 100. \qquad (154)$$

In order to find[1] the susceptibility of any other liquid all that was necessary was to suspend this glass cylinder in the liquid and read the angles on the torsion head which were required to bring the cylinder back to its zero position.

$$K_g - K_l = c\Theta$$
$$K_l - K_a = c(\Theta_a - \Theta_l) = 100\frac{\Theta_a - \Theta_l}{\Theta_a - \Theta_w}. \qquad (155)$$

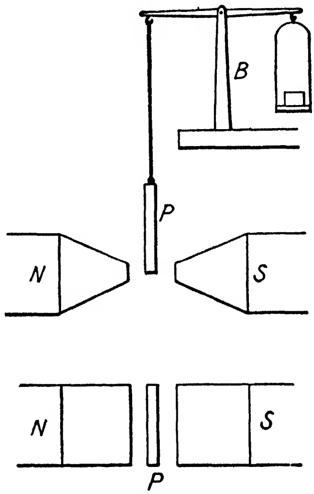


FIG. 76.—An analytical balance used in measuring the magnetic susceptibilities of substances. Gouy's method.

c. Method by Weighing.—In Sec. **34,** Fig. 66, it was shown that when the magnetic lines of force are parallel to the bounding surface between two media, the field gradient is largest in a direction normal to this bounding surface. The result is that, if the two media are free to move, the one having the largest susceptibility will attempt to displace the one with the lower value. It is substantially the problem of a body of permeability μ_2 placed in a field in a medium of permeability μ_1. The body may, for convenience, be in the form of a plate with cross-section A and permeability μ_2. This is hung with the lower edge of the plate symmetrically placed in the space between the poles of an electromagnet, as shown in Fig. 76. When the electromagnet is excited, the plate is drawn farther into the magnetic field with a given force f. This force may be measured by attaching the plate to one arm of a fine analytical balance. If the plate or cylinder is drawn into the field by a distance dx, the work done will be equal to fdx. In Eq. (73) it was shown that the energy per unit volume of a magnetic field is

$$E = \frac{\mu H^2}{8\pi}. \qquad (73)$$

[1] POYNTING and THOMSON, "Textbook of Physics," pts. 1 and 2, p. 283, 1920.

When the cylinder is displaced a distance dx the change in energy will be very closely equal to

$$dE = \left(\frac{\mu H^2}{8\pi} - \frac{H^2}{8\pi}\right)A\,dx. \tag{156}$$

If in place of air another medium μ_1 is taken, in which the cylinder is hung, a more general expression is obtained, *viz.*,

$$dE = f\,dx = \left[\frac{\mu_2 H^2}{8\pi} - \frac{\mu_1 H^2}{8\pi}\right]A\,dx, \tag{157}$$

or

$$f = \frac{H^2}{8\pi}(\mu_2 - \mu_1)A = \frac{1}{2}(K_2 - K_1)H^2A. \text{ (See Eq. [45].)}$$

The value of H has to be determined. This can be done in various ways which will be described later. This method has been employed by various investigators[1] largely because of its simplicity.

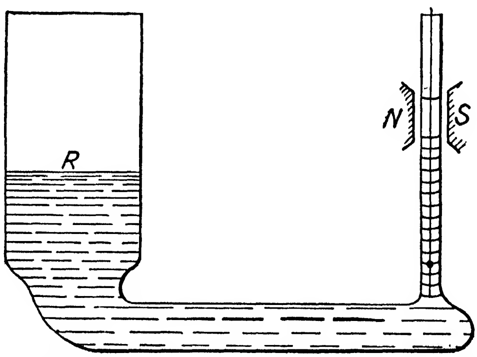


FIG. 77.—Hydrostatic balance for measuring magnetic susceptibilities.

d. Quincke's Method.—In principle this is the same as the preceding one. It has one medium displacing another one. It is more applicable to liquids than the previous one, although the former is not limited to solids.

One limb of a U-tube is placed between the poles of an electromagnet as shown in Fig. 77, or Fig. 78 when various gases are to be studied. The bore of the tube between the poles is narrow, while the other is large and is far enough outside the magnetic field so that it is not affected. Liquids are poured into the tube

[1] GOUY, *Compt. rend.*, **109**, 935, 1889;
WILLS, *Philos. Mag.*, **45**, 432, 1898;
PASCAL, *Compt. rend.*, **150**, 1054, 1910.

until the meniscus in the narrow tube stands dissymmetrically in the field. A mechanical force acts upon the meniscus as in the preceding method, and the bounding surface will rise or fall by an amount dh, such that

$$p = \rho_2 g dh = \tfrac{1}{2}(K_2 - K_1)H^2, \tag{158}$$

where K_2 is the susceptibility of the liquid, and K_1 that of the gas. It is evident at once that this method of Quincke[1] can be

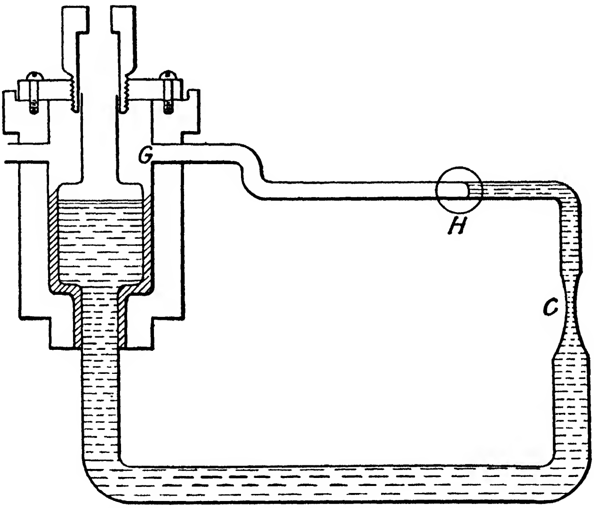


FIG. 78.—Manometric balance used by Wills and Hector. Displacement of the meniscus by the field H produces a large displacement of the colloidal particles at C. Gas G balanced against liquid.

employed for measuring[2] the strength of magnetic fields. The susceptibilities of water and air are known with sufficient accuracy to have their values substituted in Eq. (158) for the determination of H. It also follows that

$$\frac{K_2}{\rho_2} - \frac{K_1}{\rho_2} = \frac{2gdh}{H^2}$$

$$\chi_2 - \frac{K_1}{\rho_1}\frac{\rho_1}{\rho_2} = \frac{2gdh}{H^2} \qquad \text{(See Eq. [46].)}$$

$$\chi_2 = \frac{2gdh}{H^2} + \chi_1\frac{\rho_1}{\rho_2}. \tag{159}$$

[1] QUINCKE, *Wiedemann Ann.*, **24**, 369, 1885.
[2] DUBOIS, *Wiedemann Ann.*, **35**, 137, 1888.

e. The Work of Curie.[1]—The classical study in this whole field of para- and diamagnetic bodies was made by Curie. He applied Eq. (153) directly because, as he pointed out, mass susceptibility is more truly a characteristic magnetic constant than the volume susceptibility. Convenience rather than high accuracy led Curie to adopt an improved form of Faraday's torsion balance. This is shown in Fig. 79. On a torsion fiber F a sturdy U-shaped arm UX was attached and counterpoised against the index arm P by sliding small weights along P. The specimen to be studied was attached to the U-arm at X in either a glass, porcelain, or platinum receptacle. The non-uniform field was obtained by setting the two coils C and C at an angle with each other as indicated in Fig. 80. The motion of the substance in the field was very small, but by means of the microscope M the movement of X could be determined to within 1/1,000 mm.

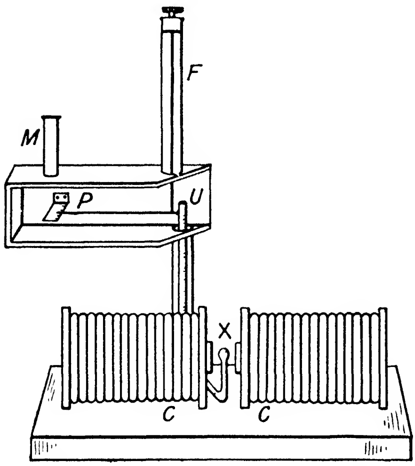


FIG. 79.—Curie's magnetic balance.

In Fig. 80 it will be noted that the emplacement of the coils and the position of the specimen are such that the point O is a point for the maximum value of HdH/dx. This is the location of the specimen for the greatest sensitivity of the balance.

Curie was concerned with the problem as to whether diamagnetism, paramagnetism, and ferromagnetism were fundamentally distinct properties. This could be studied from the effect which varying temperatures would produce on these three states. For varying the temperatures of the specimens a heating coil C (Fig. 81) was placed about the position O. As the temperature varied, the corresponding forces, acting upon the substance in the magnetic field, could be determined by means of the torsion balance. Curie's balance was standardized in terms of water for which $\chi = -0.79 \times 10^{-6}$ was taken as the absolute value for that substance. Later and more accurate values give $\chi = -0.72$

[1] CURIE, *Ann. chim. phys.*, **7**, 289, 1895.

$\times 10^{-6}$ for water, so that all of Curie's values should be multiplied by the constant $72/79 = 0.91$. Out of Curie's work it follows that diamagnetism and paramagnetism are fundamentally distinct. We shall see in Chap. VII, however, when discussing the effect of heat on magnetic properties that it is not a clean-cut distinction.

A great deal of work has been done and is being done on the *susceptibility of gases*. Of the greatest interest is the deter-

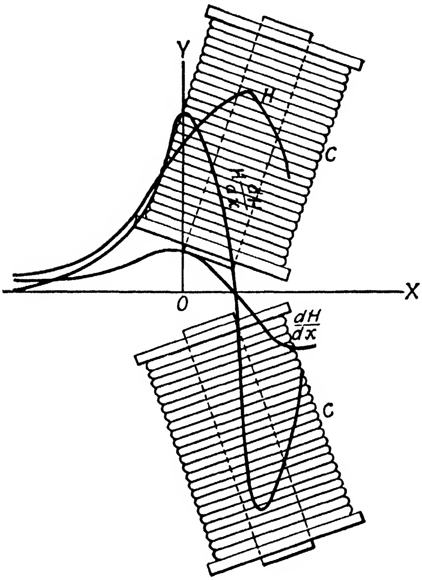


FIG. 80.—Emplacement of the specimen *O* with respect to the coils and their magnetic fields.

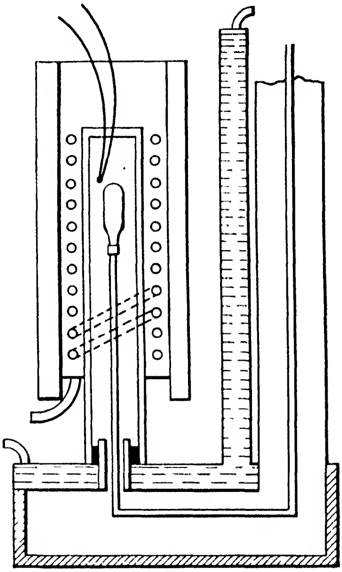


FIG. 81.—Electric furnace used with Curie's magnetic balance.

mination of the magnetic susceptibility of atomic hydrogen. According to the Rutherford-Bohr picture of the atom this should be paramagnetic. Molecular or ordinary hydrogen is diamagnetic. Wills and Hector,[1] by means of a manometric system (Fig. 78), determined the volume susceptibility of oxygen, hydrogen, and helium. The manometric balance is one in which the effect of the field on the gas is opposed to its effect on a standard solution of nickel chloride. Exact balance was obtained by varying the pressure of the gas or the common temperature.

[1] WILLS and HECTOR, *Phys. Rev.*, **23**, 209, 1924.

Hector,[1] in a later paper but using the same method, somewhat improved, determined the volume susceptibility of neon, argon, and nitrogen. The volume susceptibilities of these gases will be found in Table III.

Lehrer[2] has recently obtained some interesting results relative to the dependence of the susceptibility on pressure and temperature. An important result was that he was not able to confirm the anomalous effect in diamagnetic gases which Glaser[3] found. Vaidyanathan,[4] using a torsional balance, has carried out similar experiments on carbon dioxide and finds no confirmation of Glaser's work. Sone,[5] in an extensive research, has measured the susceptibility of hydrogen and other gases.

f. Oxley's and Pascal's Observations on Compounds.—A large amount of work has been done in studying the *susceptibilities of compounds.* In general it may be said that the magnetic susceptibility of an inorganic compound is different from the sum of the susceptibilities of the components. The additive law does not hold. Certain compounds, for instance, are diamagnetic and yet their constituents are all paramagnetic. Similarly, certain compounds are paramagnetic, although the elements composing them are diamagnetic.

A study of the magnetic properties of compounds leads to many complex relations. From these, however, may emerge results which will be of great importance in arriving at a definite theory of the nature of magnetism. This is strikingly illustrated by the work of Cabrera.[6]

A great deal of the ground work on the susceptibilities of compounds has been done by Oxley[7] and Pascal.[8] To a large extent, organic compounds are diamagnetic. Pascal concluded that "in a series of organic compounds of a similar constitution, the molecular susceptibility of a compound can be given by addition from the susceptibility of the constituent atoms or radicals."[9] If χ_m is the molecular susceptibility, then, according to Pascal,

$$\chi_m = \Sigma a \chi_a + \lambda, \tag{160}$$

[1] HECTOR, *Phys. Rev.*, **24**, 418, 1924.
[2] LEHRER, *Ann. der Phys.*, **81**, 229, 1926.
[3] GLASER, *Ann. der Phys.*, **75**, 459, 1924; **78**, 641, 1925.
[4] VAIDYANATHAN, *Indian Jour. Phys.*, **1**, pt. 2, 183, 1926.
[5] SONE, *Sci. Repts. Tôhoku Imp. Univ.*, **8**, 115, 1919.
[6] CABRERA, *Jour. Phys. et le radium*, **3**, 443, 1922.
[7] OXLEY, *Philos. Trans.*, **214** (A), 109, 1914; **215** (A), 79, 1915.
[8] PASCAL, *Ann. chim. phys.*, **19**, 5, 1910; *Compt. rend.* **173**, 145, 1921.
[9] HONDA, "Magnetic Properties of Matter," pp. 140–144.

where λ is a corrective factor, characteristic of the molecular constitution. On the whole Pascal obtains very good agreement between calculated and observed values.

Oxley and Ishiwara[1] have shown that the diamagnetic susceptibilities of organic compounds in a liquid state are nearly constant in the temperature range between 22° C. down to −192°. During solidification at low temperatures the susceptibility varies more or less discontinuously.

g. Magnetic Properties of Crystals.—Quite recently considerable attention has been paid to the magnetic properties of crystals. Inasmuch as crystalline structure seems to influence magnetic behavior it is natural to consider the effects of a magnetic field on crystals. Among the early investigators may be mentioned Tyndall,[2] whose work was largely qualitative. Voigt and Kinoshita[3] used Faraday's method and made accurate measurements on small circular disks of the crystal, cut perpendicular to one of the axes. These disks were then properly suspended between the poles of an electromagnet and measurements made when the face of the disk was perpendicular to the field.

Weiss and his pupils, in developing the electron theory of magnetism, studied the magnetic properties of natural ferromagnetic crystals with great success. Williams,[4] in "The Electron Theory of Magnetism," gives a very good description and supporting theory for the work of Weiss and his pupils. The development of the technique[5] for making single crystals of the ferromagnetic substances has made possible a very extensive study of the magnetic properties of crystals. The importance of this work cannot be overestimated. It deserves a complete treatise by itself.

Among the most recent workers in this field may be mentioned: Honda[6] and his co-laborers who have investigated particularly

[1] Ishiwara, *Sci. Repts. Tôhoku Imp. Univ.*, **3**, 303, 1914.

[2] Tyndall, "Diamagnetic and Magnecrystallic Action."

[3] Voigt and Kinoshita, *Gött. Nachr.*, p. 123, 1907.

[4] Williams, *Bull.* 62 *Univ. Ill.*, Nov. 4, 1912.

[5] Ruder, *Trans.* Amer. Soc. Steel Treating, p. 23, 1925;
Primrose, *Trans.* Amer. Soc. Steel Treating, p. 30, 1925;
McKeehan, *Sci. Monthly.* **25**, 272, 1927.

[6] Honda and Kaya, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 721, 1926;
Honda and Mashiyama, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 755, 1926;
Gerlach, *Zeitsch. für Phys.*, **38**, 828, 1926;
Williams, *Testing*, **1**, 190, 1924;
Honda, Masumoto, and Kaya, *Sci. Repts. Tôhoku Imp. Univ.*, **17**, 111, 1928.

the magnetization of single crystals of iron, nickel, and cobalt, also their magnetostrictive effects. Kapitza[1] and Webster,[2] at the Cavendish Laboratory, have done some very important work which Mahajani[3] has supported by his theoretical studies. This work on the magnetic properties of crystals, taken in conjunction with the recent work on the Barkhausen effect, is one of the important chapters in our present conceptions of magnetic phenomena.

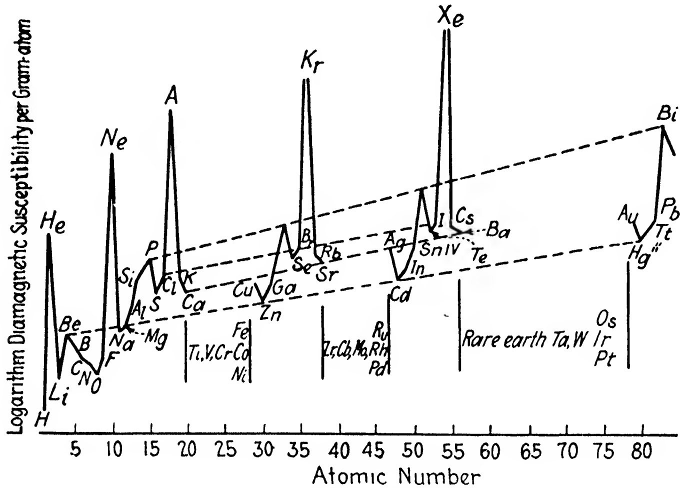


FIG. 82.—Graphical relations between atomic numbers and logarithms of magnetic susceptibilities.

Later chapters will discuss various theories of magnetism, but, in passing, it is of interest to note that most of the susceptibilities of the elements have been determined. The accuracy of some might be questioned. If one plots, as Dushman[4] has done, the logarithms of the atomic susceptibilities against the atomic numbers, one gets a curve such as that shown in Fig. 82. The striking features of this curve are that similar elements appear to lie on the same straight line, and the various lines seem to have the same slope. Figure 82 suggests the same idea conveyed by other curves in which atomic properties are plotted against atomic numbers. The curve is an argument that diamagnetism, if not magnetism in general, is an atomic phenomenon. The

[1] KAPITZA, *Proc. Roy. Soc.*, **119**, 358, 1928.

[2] WEBSTER, *Proc. Roy. Soc.*, **107**, 496, 1925; **113**, 196, 1926; **114**, 611, 1927.

[3] MAHAJANI, *Philos. Trans.*, **228**, 63, 1929.

[4] DUSHMAN, *General Electric Review*, October, 1916.

existence of a large number of anomalous observations on the magnetic behavior of substances still leaves the question an open one. Oxley, in studying the susceptibility of substances at the melting point, concludes that the diamagnetic susceptibility is not wholly an atomic property.

These important and unanswered questions make it very evident that much investigational work is still needed in this field.

CHAPTER II

MAGNETO-MECHANICS

37. Magnetostriction.—In the classification of magnetic phenomena, those which come under the head of magnetostriction form a very large group. It is that set of phenomena in which there are changes in dimensions attending magnetization and, conversely, the changes in magnetic properties accompanying mechanical stresses. To use a statement of Burrows:[1]

> Experimental evidence . . . seems to point to the conclusion that there is one and only one set of mechanical characteristics corresponding to a given set of magnetic characteristics, and conversely, there is one and only one set of magnetic characteristics corresponding to a given set of mechanical characteristics.

These related phenomena seem to be special cases of the more general principle which was stated on page xvii. It is one of the very significant features of magnetostriction that there is a reciprocal relation[2] which always exists between the deformation caused by magnetization and the change in magnetic property due to a deforming force. This means to say that if a magnetic field increases the length of a ferromagnetic rod, then a mechanical pull when applied to the same specimen will increase its permeability beyond that of the unstretched condition.

The phenomena of magnetostriction have a two-fold significance, one for pure and the other for applied science. Its importance for the field of pure science is that it must eventually yield a more comprehensive theory of atomic structure. It is a safe assertion to make that when the elementary magnet has been discovered a much better concept of the atom will be forthcoming. The atomic structure must have something to do with the phenomena of magnetostriction, and therefore the latter may

[1] Burrows, *Bull.* Bur. Stand., **13,** 207, 1916–1917.

[2] Thomson, "Application of Dynamics to Physics and Chemistry," p. 43 *et seq.*, 1888;

Williams, *Proc.* Amer. Soc. Testing Materials, **19,** pt. 2, 1919.

be expected to yield some supplemental conception of the architectural design of the atom.

From the practical standpoint magnetostriction has a contribution to make. It seems quite probable from what has already been done that it will be possible to study the mechanical properties of steel, nickel, cobalt, and other ferromagnetic substances by means of their magnetic behavior.[1] If from ordinary magnetic properties it is possible to arrive at mechanical properties, surely the magnetostrictive effects, in which magnetic and mechanical properties always play such an intimately associated rôle, must contribute greatly to our knowledge of magneto-mechanical analysis. This phase of the subject should be of immense value to the steel industry where it has already been shown that magnetic analysis is quite dependable for picking up deep-seated flaws. There are two methods[2] at present, practically the only two, for testing the homogeneity of a substance without destroying the specimen. These are the magnetic and x-ray methods of analysis. To be able to detect the presence or absence of flaws in a steel member of any structure *in situ*, or before being set in place, or before being machined, would be a contribution of the first order. Such a proposition does not seem impossible.

For the most part the magnetostrictive effects have to do with dimensional changes involving length, twist, and volume. Since the phenomena of magnetostriction form so large a part of general magnetic phenomena it is important that the method and technique of research in this field should be set forth and attention called to the numerous problems which remain unsolved.

38. Mechanical Strains Due to Magnetic Stresses.—*a. Linear Changes.* 1. *Joule Effect—Change in Length Due to a Longitudinal Magnetic Field.*—Let a rod of iron or other ferromagnetic substance be placed in a solenoid as in Fig. 83. When an electric current flows in the coil, the rod becomes magnetized

[1] PAGET, "The Engineer," p. 463, Nov. 29, 1867;
McCANN and COLSON, *West. Electr.*, **43**, 76, 1908;
BURROWS, *Bull.* 272 Bur. Stand., p. 173, 1916;
SANFORD and KOUWENHAVEN, *Sci. Paper* 343 Bur. Stand., October, 1919;
SANFORD and FISHER, *Proc.* Amer. Soc. Testing Materials, **76**, 65, 1919;
WILLIAMS, *Jour.* Cleve. Eng. Soc., p. 183, January, 1917;
SUZUKI, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 479, 1926.

[2] *Elec. Times*, **56**, 131, 1919.

longitudinally and changes its length. This change in length is a very small quantity and, in order to measure it accurately, extensometers must be used which magnify the movements produced by these changes in length.

Joule[1] first discovered the changes in length of an iron rod when magnetized and found it to be an increase. His extensometer consisted of two levers of the first class in tandem. The motion

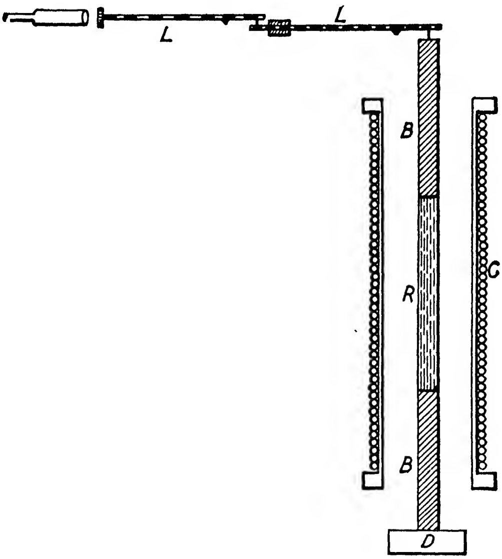


FIG. 83.—Extensometer used by Joule in measuring the changes in length of of an iron rod when magnetized.

of the outer end of the second lever was amplified by means of a microscope. Figure 83 shows this schematically. The idea of a mechanical lever attached to the end of the rod, whose change in length is to be measured, has been common to a number of extensometers.[2] This mechanical lever has, in turn, been supplemented by an optical lever of some sort. The combination of

[1] JOULE, *Philos. Mag.*, **30**, 76, 225, 1847.

[2] MORE, *Philos. Mag.*, **40**, 345, 1895;
BRACKETT, *Phys. Rev.*, **5**, 275, 1897;
HONDA and SHIMIZU, *Jour. Coll. Sci. Imp. Univ. Tokyo,* **19**, 1, 1903;
AUSTIN and GUTHE, *Bull.* Bur. Stand., **2**, 297, 1906;
HONDA and KIDO, *Sci. Repts. Tôhoku Imp. Univ.*, **9**, 221, 1920;
WILLIAMS, *Phys. Rev.*, **34**, 258, 1912; *Jour.* Opt. Soc. Amer., **7**, 1011, 1923;
HONDA and MASHIYAMA, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 754, 1926;
MCKEEHAN and CIOFFI, *Phys. Rev.*, **28**, 146, 1926.

the two is very well illustrated by the outfit used by Bidwell.[1] This is shown in Fig. 84. Bidwell extended the work of Joule and showed that in the case of iron the increase in length attained a maximum at a certain field strength beyond which the rod retracted and finally became shorter than in the unmagnetized state. As may be seen in Fig. 84, Bidwell worked with his iron in the form of a ring. From the change in diameter as the magnetizing current was increased it was possible to get the coefficient of lengthening.

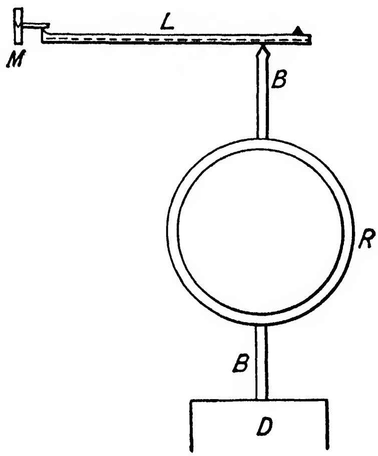


FIG. 84.—Bidwell measured the changes in length of steel by measuring the changes in diameter of a toroidal electromagnet when it was magnetized.

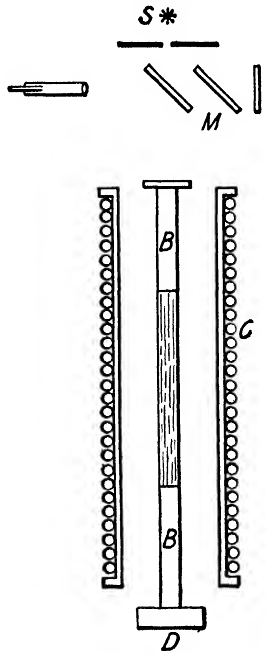


FIG. 85.—Schematic arrangement of the Michelson interferometer for measuring the changes in length due to a magnetic field.

The development of the interferometer led to its use in measuring changes in length. The Michelson arrangement of mirrors for this purpose is shown in Fig. 85. Lochner[2] and Stevens[3] used the interferometer in their measurements. Newton's rings[4] may be employed for the same purpose.

A complete departure from optical means in measuring the Joule magnetostrictive effect is found in the extensive work of

[1] BIDWELL, *Proc. Roy. Soc.*, **55**, 228, 1894; **56**, 94, 1894.

[2] LOCHNER, *Philos. Mag.*, **36**, 498, 1893.

[3] STEVENS, *Phys. Rev.*, **7**, 19, 1898.

[4] WILLIAMS, *Phys. Rev.*, **4**, 500, 1914.

Schulze.[1] Schulze employed Whiddington's[2] ultramicrometer. One plate of a very small condenser is attached to the upper end of the rod whose length is to be varied. As the rod elongates or contracts, the distance between the plates varies and, therefore, the capacity varies also. This changes the natural frequency of the circuit of which the condenser is a part, and by a beat method it is possible to observe a small change in the frequency of the capacity which, in turn, measures the change in the length of the rod. Whiddington claims an accuracy of 10^{-8} cm. with his outfit. McKeehan and Cioffi could measure 4 times 10^{-8} cm. per division on their galvanometer scale. Honda found that with his outfit for measuring changes in length, one millimeter division of deflection represented a change in length of 4.5 times 10^{-8} cm.

Since in the Joule effect the changes in length are so very small the utmost care must be maintained in order that no outside factors will produce changes which would blanket the effects sought for. Temperature changes, in particular, must be avoided and every means used to maintain a uniform temperature.[3] Uniform magnetic fields should be employed as well as the means for testing their uniformity.[4] If the specimen is in the form of a rod and is magnetized by a solenoid, the rod should be short enough so that its ends fall far within the solenoid. The rod should also be placed symmetrically in the field, otherwise there would be a mechanical traction on the specimen which would vitiate the results. The solenoid and the rod should be mounted as independently of each other as possible. The method of supporting the specimens on the coil itself is to be avoided.[5] All extraneous magnetic fields, such as the earth's field, fields of ammeters, and electromagnets, must be eliminated. This is particularly true for nickel which is hypersensitive to stray fields. The results for nickel are quite different when the earth's field is compensated from what they are when the earths' field is not annulled.[6] All test samples must be completely demagnet-

[1] Schulze, *Zeitsch. der Phys.*, **50**, 448, 1928.

[2] Whiddington, *Philos.*, *Mag.*, **40**, 634, 1920.

[3] Nagaoka, *Philos. Mag.*, **37**, 132, 1894;
Williams, *Phys. Rev.*, **34**, 259, 1912.

[4] Williams, *Jour. Franklin Inst.*, **182**, 353, 1916.

[5] Williams, *Phys. Rev.*, **32**, 285, 1911.

[6] Honda and Shimizu, *Wiedemann Ann.*, **14**, 793, 1904;
Heydweiler, *Ann. der Phys.*, **15**, 415, 1904;
Williams, *Phys. Rev.*, **10**, 135, 1917.

ized before a set of readings is taken. It must be done in a space free from extraneous fields. If not, nickel, for instance, when demagnetized will show a polarity in the same direction as the extraneous field.

Whether for controlling the alternating current in the demagnetizing process, or the direct current for magnetizing the specimen, a liquid rheostat in series with the solenoid is to be especially recommended.[1] For currents up to 5 to 7 amperes they work very well when the current is on for a short time.

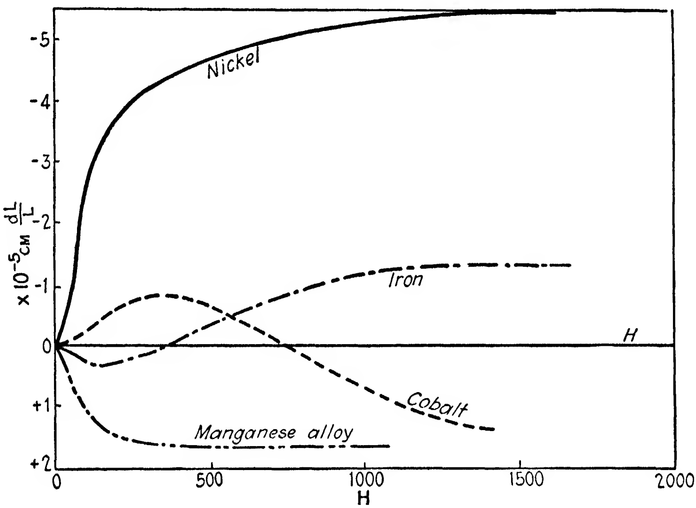


FIG. 86.—This graph illustrates the way in which various ferromagnetic substances change their length in a magnetic field. All values above the zero line represent contraction and all below indicate an elongation of the specimen.

There are various conditions under which the Joule effect may be studied. It may be studied when the specimens are under tension or under compression. The effect varies greatly with change of temperature. In our present state of knowledge one can hardly speak of the Joule effect under normal conditions, but for the conditions under which the Joule effect is ordinarily studied, *i.e.*, at room temperature and the specimen freely suspended in the magnetizing coil, it is of interest to compare the effect in the various ferromagnetic bodies. Figure 86 shows four fairly typical curves for iron, nickel, cobalt, and an alloy of manganese. Iron increases its length for weak fields and decreases it for strong. Nickel shortens for all field strengths, while the

[1] WILLIAMS, *School Sci. and Math.*, **12**, 489, 1912.

alloy of manganese, known as the Heusler alloy, lengthens for all field strengths. Cast cobalt, on the other hand, is almost the opposite of that for iron in that it shortens for small magnetizing forces and lengthens for strong. In all of these curves a field strength is finally reached where further increase in the magnetizing force produces no change in length.

Just as there is hysteresis in the process of magnetization, so there is hysteresis attending the change in length produced by magnetization. Knott, Nagaoka, and Williams[1] have made observations on these magnetic-elongation cycles. The same equipment is needed for this work as for the regular Joule phenomenon. It is also possible, according to McCorkle,[2] to study the Joule effect under anhysteretic conditions.

Because of hysteresis, the change in length which accompanies magnetization, due to an alternating field, must be different from that for a steady field. This is indicated by the work of Nagaoka and Honda, and Austin, and Brown.[3] Buckley and McKeehan[4] have shown a very close relationship between the hysteresis loss and the magnetostrictive effect in permalloy, an alloy of iron and nickel, having a very high initial permeability. Wwedensky and Simanow[5] find a corresponding relation in nickel alone. That this relationship is not universal seems to follow from some work done by the author[6] on a series of nickel rods of different degrees of hardness.

A study of the Joule magnetostrictive effect in single crystals is highly interesting and appears to offer a clearer insight into the behavior of rods of the same material in which the small crystals composing the sample are oriented at random. Heaps[7] worked on this effect in magnetite and brought out many of the salient features of this phenomenon in a ferromagnetic crystal. His paper has not received the attention it deserves. Williams,

[1] Knott, *Philos. Mag.*, **37**, 141, 1894;
Nagaoka, *Philos. Mag.*, **37**, 131, 1894;
Williams, *Phys. Rev.*, **34**, 265, 1912.

[2] McCorkle, *Phys. Rev.*, **25**, 541, 1925.

[3] Nagaoka and Honda, *Philos. Mag.*, **4**, 64, 1902;
Austin, *Phys. Rev.*, **10**, 180, 1900;
Brown, *Proc. Roy. Soc. Dublin*, ser. of pap., 1914–1916.

[4] Buckley and McKeehan, *Phys. Rev.*, **26**, 261, 1926.

[5] Wwedensky and Simanow, *Zeitsch. für Phys.*, **38**, 202, 1926.

[6] Williams, *Science*, **65**, 306, 1927.

[7] Heaps, *Phys. Rev.*, **24**, 60, 1924.

Webster, Honda and Mashiyama, and McKeehan[1] have made contributions to this subject which have thrown a great deal of light on the mechanism of the effect. McKeehan's theoretical considerations deserve especial attention.

2. *Transverse Joule Effect—Change in Dimensions Normal to the Magnetic Field.*—The first effect discovered by Joule[2] in magnetostriction led him to wonder if a volume change occurred simultaneously with the change in length. Due to lack of sensitivity of apparatus he was not able to find it, so looked for a change in dimensions normal to the field in order to compensate for the change in length. This effect he found and is now known as the transverse Joule effect. Joule's first method was to run an electric current in an insulated wire inside of an iron tube. This gave circular lines of force, and the change in dimensions, normal to the field, could be observed by measuring the change in length of the tube.

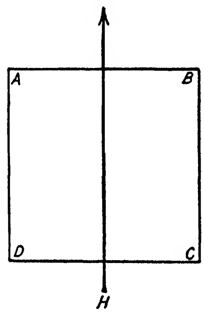


Fig. 87.—A comparison of the longitudinal and transverse magnetostrictive effects discovered by Joule.

Further investigations of this change in dimensions have been made by Bidwell, Williams, Brown, and Heaps.[3] There are two ways in which one may compare the Joule transverse with the Joule longitudinal effect. Let $ABCD$ (Fig. 87) be the cross-section of a cube of iron. If a magnetic field H is imposed on the cube in the direction of the arrow, measurements along AD and CD would constitute observations for the longitudinal and transverse effects respectively. In this case the direction of the magnetic field is kept constant and the measuring device is shifted. On the other hand, if the field H is first applied parallel to AD and its change in dimension observed, that would be the longitudinal effect. But if the magnetic field should be applied parallel to CD, any change in length along AD would be

[1] Williams, *Testing*, **1**, 190, 1924;
Webster, *Proc. Roy. Soc.*, **109**, 570, 1925;
Honda and Mashiyama, *Sci. Repts. Tôhoku Imp. Univ.*, **15**, 755, 1926;
McKeehan, *Jour. Franklin Inst.*, **202**, 737, 1926.

[2] Joule, *Philos. Mag.*, **30**, 225, 1847.

[3] Bidwell, *Proc. Roy. Soc.*, **56**, 94, 1894;
Williams, *Phys. Rev.*, **4**, 498, 1914;
Brown, *Proc. Roy. Soc. Dublin*, **15**, 212, 1916;
Heaps, *Phys. Rev.*, **6**, 34, 1915.

a transverse effect. In this second case, the direction of the magnetic field has been shifted instead of the measuring apparatus.

As a rough approximation it might be said that the transverse and longitudinal effects are opposite, *i.e.*, when the elongation occurs longitudinally, contraction occurs transversely and *vice versa*. The opposite effects are particularly true for crystals. Inhomogeneity and the fact that there is a volume change as well prevents an exact reversal of the two effects for ordinary specimens.

3. *Guillemin Effect—Bending Due to a Magnetic Field.*—The literature on this subject is very meagre and for the most part of ancient lineage. Guillemin[1] observed that an iron rod when slightly bent would tend to straighten on the application of a longitudinal magnetic field. He used a rod one centimeter in diameter and about 25 cm. long which was clamped at one end and bent down by a small weight attached to the free end. A solenoid surrounded the rod coaxially. When the field was applied, the free end of the rod was lifted a little. Wertheim[2] investigated this same problem and studied especially the effect of the rod not being coaxial with the magnetizing coil. This would naturally make a difference in the effect. Honda, Shimizu, and Kusakabe[3] obviated any trouble due to this cause by winding the magnetizing coil directly upon the rod, so that, as the rod bent, the coil went with it. Undoubtedly the best work which has been done on this phase of the subject is contained in the paper just mentioned. Stevens and Dorsey[4] have studied the change in elastic constants of steel by this method and obtained positive results. Their method involved a clever application of the Michelson interferometer and is shown in Fig. 88. Recently Miller[5] presented a paper on magnetostriction in which a horizontal wire is anchored at both ends and the sag, when the wire is longitudinally magnetized, is a measure of the change in length of the wire. This is in part a phase of the Guillemin effect. The effect, however, is so involved that it is difficult to separate the essential features. Guillemin's effect has the unfortunate condition that the rod or wire in a horizontal

[1] GUILLEMIN, *Compt. rend.*, **22**, 264, 432, 1846.

[2] WERTHEIM, *Compt. rend.* **22**, 336, 1846; *Poggendorff Ann.*, **68**, 140, 1846; *Ann. chim. phys.*, **23**, 302, 1848.

[3] HONDA, SHIMIZU, and KUSAKABE, *Philos. Mag.*, **4**, 459, 1902.

[4] STEVENS and DORSEY, *Phys. Rev.*, **9**, 116, 1899.

[5] MILLER, *Proc.* Amer. Phys. Soc. Meet., Washington, D. C.

position has the upper half in a state of tension while the lower half is compressed (Fig. 89). When the rod shortens or lengthens, according to the Joule effect, does this differential effect cause the bending or sagging, or is there a tendency for the longitudinal fibers of the bar or wire to set themselves parallel to the

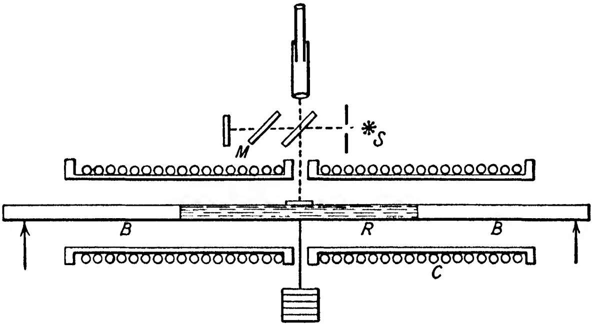


FIG. 88.—Weights *W* were added to bend the bar *R*. The application of a magnetizing force parallel to the rod caused the interference fringes to shift in the field of view.

field? In the effect of a permanently twisted rod, seeking to untwist when longitudinally magnetized, it would appear that the second must be present. Doubtless the first is also to some extent. In our present state of knowledge concerning this effect the Guillemin method doesn't seem the most logical way of getting at the general principle which underlies magnetostriction.

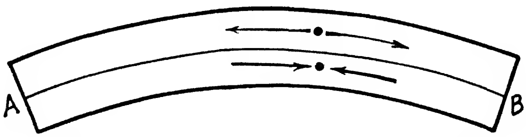


FIG. 89.—In the Guillemin effect, part of the rod is stretched and the remainder is compressed.

4. *Changes in Young's Modulus Due to Magnetization.*—If one gives equal weight to the earlier and later papers on this subject they would be very confusing. Wertheim and Tomlinson[1] were very sure that there was no effect of a magnetic field on the modulus of stretch. Brackett[2] found an increase of E amounting to one-half per cent. In some experiments on the

[1] TOMLINSON, *Proc. Roy. Soc.*, **40,** 447, 1886; **47,** 13, 1889; WERTHEIM, *Ann. chim. phys.*, **12,** 610, 1842.

[2] BRACKETT, *Phys. Rev.*, **5,** 257, 1897.

bending of rods, due to a magnetic field, Stevens and Dorsey[1] found an increase in E both for wrought iron and for steel. Later Stevens[2] confirmed this increase of E by stretching experiments which is a more direct method. The most convincing and thorough-going work was done by Honda, Shimizu, and Kusakabe and Honda and Terada,[3] in which they found an increase in the modulus of stretch due to a magnetic field for iron, steel, and cobalt.

The elastic constant of a substance in a magnetic field is to be defined as the ratio of the stress applied, to the strain caused thereby, the magnetic force constantly acting on the substance. The change of elasticity is then the difference of this quantity when the magnetic field is on and when it is off.

Because of hysteresis much of the former work on this subject has no significance because the right order of applying the field and the stress was not observed. Nickel showed a decrease of E for weak fields and an increase in strong fields. The work of the last-mentioned investigators seems to establish pretty thoroughly the positive character of the effect. Their methods for measuring the changes in the modulus was by the ordinary means employed in the laboratory. Further work on this problem should seek to develop greater refinements in the present methods of measuring Young's Modulus.

b. Circular Changes. 1. *Wiedemann Effect—Twist Due to the Interaction of Circular and Longitudinal Fields.*—If simultaneously a circular and a longitudinal field are impressed on a rod of iron or other ferromagnetic substance the resultant field is directed along a helix about the rod (Fig. 90). If a change in length occurs along this resultant field a twist of the rod ensues. This is what Wiedemann[4] discovered, *viz.*, that, if an electric current is sent through the rod, producing a circular field, and at the same time the rod is magnetized longitudinally, a twist occurs.

The character of the effect is such that it agrees, qualitatively at least, with the phenomena observed in the Joule effect

[1] Stevens and Dorsey, *Phys. Rev.*, **9**, 119, 1899.

[2] Stevens, *Phys. Rev.*, **11**, 95, 1900.

[3] Honda, Shimizu, and Kusakabe, *Philos. Mag.*, **4**, 459, 1902; Honda and Terada, *Philos. Mag.*, **13**, 36, 1907; **14**, 65, 1907; Brown, *Proc. Roy. Soc. Dublin*, **15**, 185, 1917.

[4] Wiedemann, "Lehre von der Elektrizität," vol. III, p. 680 *et seq.*, 1883.

and makes it a special case of Joule's discovery.[1] The difficulty in getting quantitative relations is in ascertaining correctly the distribution of the circular magnetic field. It is somewhat better in the case of the Wiedemann effect in thin-walled tubes where the circular field is produced by a current through a

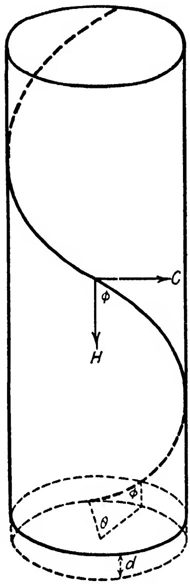


FIG. 90.—A longitudinal and circular magnetic field impressed simultaneously give a resultant helical field.

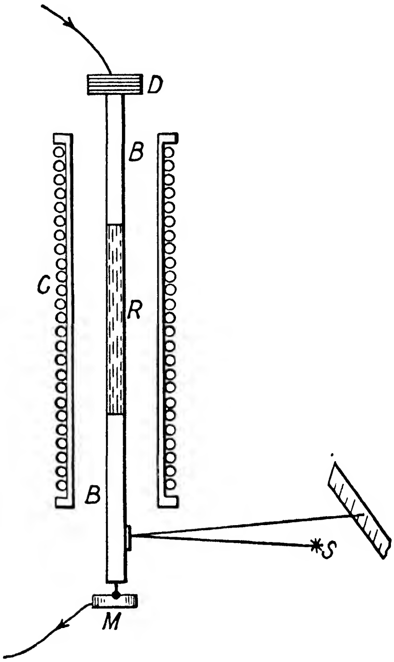


FIG. 91.—The mechanical twist due to superimposing a longitudinal and a circular magnetic field upon a ferromagnetic rod is easily measured by means of a telescope and scale.

wire, which runs inside of the tube but is insulated from it. The twist is observed by simply attaching a mirror to the end of the rod or tube which is not fixed (Fig. 91). No amplification of the twist is needed as it may easily be measured directly with telescope and scale. The Wiedemann effect may be studied in three different ways:[2] (1) Holding the longitudinal field constant

[1] WILLIAMS, *Phys. Rev.*, **32**, 281, 1911;
FROMY, *Jour. der Phys.*, **7**, 13, 1926.

[2] KNOTT, *Philos. Mag.*, **30**, 244, 1890;
NAGAOKA and HONDA, *Philos. Mag.*, **4**, 45, 1902;
HONDA and SHIMIZU, *Philos. Mag.*, **5**, 650, 1903;
WINKELMANN, "Handbuch der Physik," vol. V, p. 323, 1908;
PIDGEON, *Phys. Rev.*, **13**, 209, 1919.

and varying the circular field; (2) Holding the circular field constant and varying the longitudinal field; (3) Varying both fields simultaneously. The Wiedemann effect has been the subject of a great many investigations.

2. *Twisting of Rods Having Permanent Torsional Set When Magnetized Longitudinally or Circularly.*—Wiedemann, Smith, Groesser, and Williams[1] have observed that, if a rod is given a permanent torsional set, a longitudinal magnetic field will cause it to untwist and the direction of the imposed magnetic field has nothing to do with the direction of the untwisting of the rod. It would appear that the filaments of metal along the direction of permanent set tend to straighten out parallel to the imposed field and so untwist the rod. This is a difficult effect to study because it is impossible to obtain homogeneous bars with uniform twist. It needs further study, however, and supposedly the rod with a permanent twist ought to have a torque put upon it when magnetized circularly. These effects may be observed in the same way as the Wiedemann effects.

3. *Change in the Coefficients of Rigidity Due to a Magnetic Field.*—The earlier work on this subject is contradictory. Wiedemann[2] observed that the torsion of an iron bar or wire was diminished by magnetization. Kimball[3] claimed that he found an increase in rigidity for iron amounting to 0.9 per cent at the maximum magnetization to which he took the material. Of all the elastic moduli, the coefficients of rigidity seem to be affected the most by magnetization. Among the various investigators, Tomlinson, Barus, Day, Stevens, Honda, Shimizu, Kusakabe, and Terada,[4] there seems to be this agreement that there is an increase in the coefficient of rigidity of iron. The last-named workers found that not only iron but also steel and cobalt increased their rigidity with magnetization, while nickel showed a decrease.

[1] WIEDEMANN, *Poggendorff Ann.* **103**, 571, 1858;
SMITH. *Philos. Mag.*, **32**, 385, 1891;
GROESSER, "Dissertation," Rostock, 1896;
WILLIAMS, *Amer. Jour. Sci.*, **36**, 555, 1913.

[2] WIEDEMANN, "Lehre von der Elektrizität," vol. III, p. 796, 1883.

[3] KIMBALL, *Amer. Jour. Sci.*, **18**, 99, 1879.

[4] TOMLINSON, *Proc. Roy. Soc.*, **40**, 447, 1886;
BARUS, *Amer. Jour. Sci.*, **34**, 175, 1887;
DAY, *Electrician*, **39**, 480, 1897;
STEVENS, *Phys. Rev.*, **10**, 161, 1900;
HONDA, SHIMIZU, and KUSAKABE, *Philos. Mag.*, **4**, 537, 1902;
HONDA and TERADA, *Philos. Mag.*, **13**, 75, 1907; **14**, 65, 1907.

In all of the investigations on the changes of the elastic moduli in the future it would be desirable to refine the methods employed for studying the elastic constants.

When the technique for producing a single long, slim crystal is more fully developed it would be particularly interesting to study all of the magnetostrictive effects dealing with circular changes in a single crystal.

c. Volume Changes. 1. *Barrett Effect—Change in Volume Due to a Magnetic Field.*—This is the most elusive of all the changes in dimensions which are produced by a magnetic field. The early investigators doubted its presence. Joule[1] looked for a change in volume by placing the iron in a tube which terminated in a capillary. The piece of iron was surrounded by a liquid and any change in volume would have changed the position of the meniscus of the fluid in the capillary. No such movement occurred. This was due in part to the fact that the specimen was not magnetized to the point of saturation.

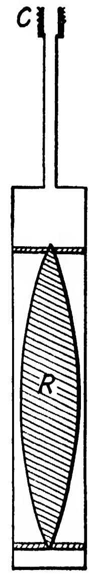


FIG. 92.—Magnetizing the ferromagnetic body *R* changes its volume. Hydrostatically compressing the ferromagnetic body *R* changes its magnetic properties.

Historically, Barrett[2] was the first to detect a slight change in volume due to a magnetic field. He used nickel as the material for observation and his results are qualitative. Bidwell[3] gives Knott[4] the credit for first detecting a change in volume due to magnetization, but Knott's work was with hollow iron, steel, and nickel tubes in fields that were not uniform and, therefore, his results are questionable. The best results were obtained by Nagaoka and Honda,[5] who worked with a dilatometer containing the specimen in an ovoid form which would give uniform distribution of the magnetic flux (see Fig. 92).

2. *Change in Bulk Modulus Due to a Magnetic Field.*—This is not a well-established phenomenon, as it is extremely difficult to measure so small a quantity. The work of Bock[6] bears upon this subject. The evidence for the existence of such an effect rests upon the change in the stretch and rigidity

[1] JOULE, *Philos. Mag.*, **30**, 225, 1847.

[2] BARRETT, *Nature*, **26**, 585, 1882.

[3] BIDWELL, *Proc. Roy. Soc.*, **56**, 99, 1894.

[4] KNOTT, *Proc. Roy. Soc. Edinb.*, **19**, 85, 1891–1892.

[5] NAGAOKA and HONDA, *Philos. Mag.*, **4**, 56, 1902.

[6] BOCK, *Wiedemann Ann.*, **54**, 442, 1895.

moduli and that there is a change in volume due to a magnetic field. The best evidence available is indirect. Direct experimental evidence is needed on this subject.

d. General Considerations.—There is great need for a comprehensive theory covering the problem of deriving theoretical relationships between the strength of the magnetizing field and the consequent strains of the magnetic medium. Maxwell, Helmholtz, Kirchhoff, Cantone, Nagaoka and Honda, Houston, and Sano[1] have made attempts at this problem, and while a fair qualitative relation has been developed, they all leave much to be desired in the way of quantitative agreement between theory and experiment. This is a field of research needing the attention of the theoretical physicist.

39. Magnetic Changes Due to Mechanical Stresses.

a. Linear Changes.—1. *Villari Effect—Change in Induction Due to Longitudinal Stresses.*—Matteucci's[2] name is the first to appear in connection with this subject. He discovered that pulling or stretching a ferromagnetic rod changes its magnetic induction. Villari,[3] however, with greater refinement of method found that if an iron rod is stretched when weakly magnetized its magnetic induction will be increased, whereas stretching the same rod in a strong field its magnetic induction will be decreased. This is known as the Villari reversal effect and the point V (Fig. 93), where stretching (or compression) does not affect the intensity of magnetization is called the Villari reversal point. The Villari effect is just the reciprocal of the Joule effect. Taking iron as an example, there is first an increase in magnetization and then a decrease as the field increases from zero upwards, corresponding to the Joule effect in the same specimen wherein the rod will first lengthen and then shorten, as the field is increased from zero upwards. As in the Joule effect, the previous history of the rods, heat treatment, mag-

[1] MAXWELL, "Electricity and Magnetism," 2d ed., p. 257;
HELMHOLTZ, *Wiedemann Ann.*, **13**, 385, 1881;
KIRCHHOFF, *Sitz. ber. der K. Akad. der Wissensch.*, Berlin, p. 47, 1884;
CATONE, *Mem. d. R. Accad. de Lincei*, **6**, 487, 1890;
NAGAOKA and HONDA, *Philos. Mag.*, **46**, 261, 1898; **49**, 329, 1900; **4**, 45, 1902;
HOUSTON, *Philos. Mag.*, **21**, 78, 1911;
SANO, *Phys. Rev.*, **14**, 158, 1902.

[2] MATTEUCCI, *Ann. chim. phys.*, **53**, 416, 1858.

[3] VILLARI, *Poggendorff Ann.*, **126**, 87, 1868.

netization, stresses, etc., will, to a large extent, determine the character of the Villari effect. Any of the methods for testing

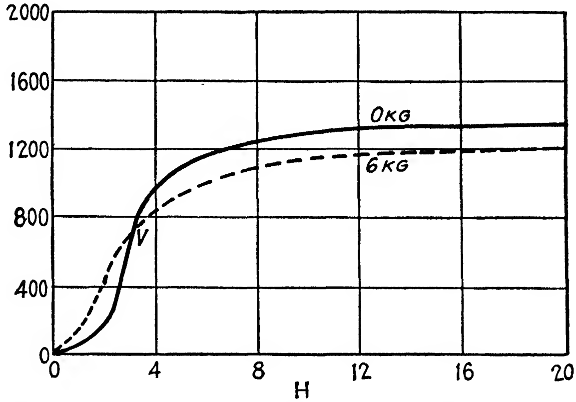


FIG. 93.—Stretching an iron rod longitudinally increases the intensity of magnetization in weak fields and decreases it in strong fields.

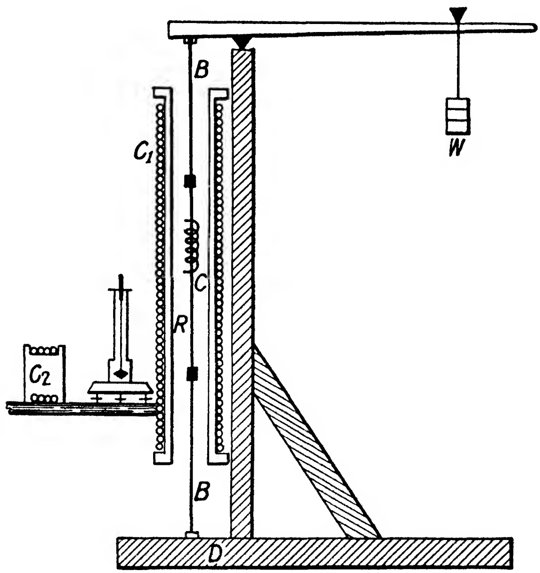


FIG. 94.—An outfit for measuring simultaneously the Villari magnetostrictive effect, ballistically and magnetometrically.

the magnetic properties of a substance are applicable for studying the Villari effect. For absolute measurements the ballistic methods are to be recommended, but for comparative work the

magnetometric method commends itself. It is a very satisfactory method to get quantitative values with the ballistic method on the normal rod and then study relative values by means of the magnetometer when the rod is stretched and again when in the normal condition. Figure 94 shows a combination of the two methods employed in studying the Villari effect in long, slim rods.

2. *Transverse Villari Effect—Change in Induction Due to Transverse Stresses.*—The reciprocal of the transverse Joule magnetostrictive effect has had little or no study put upon it. It could be studied very easily. This effect is the change in magnetic induction of a body in a direction normal to that in which a mechanical elongation or compression is applied. If a rod of iron, for instance, was compressed normal to its length what would be the change in the magnetic flux longitudinally?

3. *Converse of the Guillemin Effect—Change in Induction Due to Bending.*—To every magnetostrictive effect there is a reciprocal effect. If this is true, then there must be a converse of the Guillemin effect. If a bent rod of steel tends to straighten under the influence of a longitudinal magnetic field, then bending the same rod in a longitudinal magnetic field should change its magnetic flux. This is another one of those effects on which there has been little or no research work done. It is obvious in this, as in the Guillemin effect itself, that there is a number of factors which it would be difficult to separate if a study were made of this phenomenon.

b. Circular Changes. 1. *Inverse Wiedemann Effect—Longitudinal Magnetization Due to Twisting a Circularly Magnetized Rod.*—Wiedemann,[1] in investigating the second effect which bears his name, proceeded along lines similar to Wertheim.[2] A steel rod was placed in a torsion lathe (Fig. 95) with a long coil coaxial with the rod and connected to a ballistic galvanometer. To produce the circular magnetization an electric current flowed lengthwise of the rod. When the rod, circularly magnetized, was suddenly twisted a transient current was produced in the coaxial coil, indicating that the rod had been magnetized longitudinally. Attention is called to a discussion of these reciprocal effects and a theory of the same given by Honda.[3]

[1] WIEDEMANN, "Lehre von der Elektrizität," vol. III, p. 680 *et seq.*

[2] WERTHEIM, *Compt. rend.* **35**, 702, 1852.

[3] HONDA, "Magnetic Properties of Matter," p. 73, 1928; KOBAYASI, *Jap. Jour. Phys.*, **5**, 1, 1928.

2. *The Wertheim Effect—Circular Magnetization Due to Twisting a Longitudinally Magnetized Rod.*—A steel rod is placed in a torsion lathe as in Fig. 95 and magnetized longitudinally by the surrounding solenoid. The terminals of a ballistic galvanometer are connected to the ends of the rod. When this longitudinally magnetized rod is suddenly twisted a deflection of the galvanometer occurs, indicating an electric current along the rod and that a circular magnetization of the rod has been produced.[1]

Wiedemann[2] has studied very extensively not only the two effects found by him but also the one discovered by Wertheim.[3] Deservedly all three should be called Wiedemann effects.

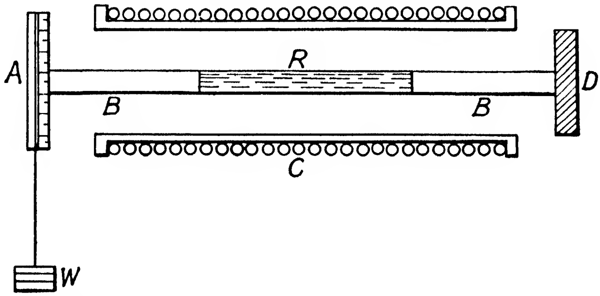


FIG. 95.—Twisting a circularly magnetized rod gives rise to a longitudinal magnetization. Twisting a longitudinally magnetized rod gives rise to a circular magnetization.

3. *Obtaining Longitudinal and Circular Fields by Twisting Rods with a Permanent Torsional Set.*—If magnetizing a rod with a permanent torsional set gives rise to a mechanical twisting of the rod, then we should expect that reciprocally a mechanical twisting of a permanently twisted rod would give rise to both a longitudinal and a circular magnetization. This has not yet been observed.

c. Volume Change. Nagaoka-Honda Effect—Change in Intensity of Magnetization Due to Volume Change.—Here again the splendid work of Nagaoka[4] and his pupils has correlated these magnetic and mechanical characteristics of this effect with those given under the Barrett effect. Just as the Villari effect is the reciprocal of the Joule effect, so also is the Nagaoka-Honda effect a reciprocal of the Barrett phenomenon. In the first relation it

[1] NAGAOKA and HONDA, *Jour. Coll. Sci.*, **13**, 263, 1900.

[2] WIEDEMANN, "Lehre von der Elektrizität," vol. III, p. 680 *et seq.*

[3] WERTHEIM, *Compt. rend.*, **35**, 702, 1852.

[4] NAGAOKA and HONDA, *Philos. Mag.*, **46**, 261, 1898.

is a matter of length, and in the second a question of volume. Like its reciprocal effect, the change in intensity of magnetization, due to a change of volume, is very small and extreme care must be exercised in its measurement. Very recently, under the direction of Professor Bridgman, Mr. Yeh[1] has carried out some very careful experiments on this phase of magnetostriction in which much higher pressures were used than by Nagaoka and Honda.

d. General Considerations.—It must be obvious that a number of problems remain to be solved in the general field of magnetostriction. It is a particularly interesting field and the concepts derived from such studies must eventually help in the general picture of atomic structure and arrangement. McKeehan's[2] recent work is a step in the right direction.

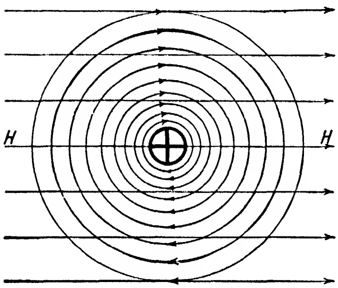


Fig. 96.—If a circular magnetic field about a straight conductor is combined with a uniform magnetic field at right angles to the conductor, there arises a resultant field shown in Fig. 97.

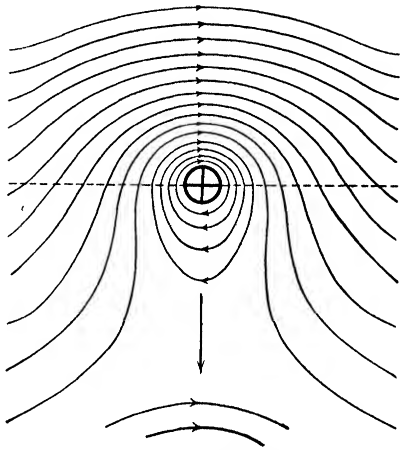


Fig. 97.—The resultant of the two magnetic fields shown in Fig. 96 produces a mechanical action on the conductor.

If a general suggestion were to be made, it would be that in future research work in this field, more attention be paid to refinement of method and to the purity of materials. Wherever possible one should use a single crystal as the specimen.

40. Forces between Magnetic Fields.—There are certain mechanical reactions between magnetic fields which are rather

[1] Yeh, *Proc.* Amer. Acad. Arts Sci., **60,** 503, 1925.

[2] McKeehan, *Jour. Franklin Inst.,* **202,** 737, 1926.

fundamental to our conceptions of magnetic phenomena. We talk about Coulomb's law for the forces acting between magnetic poles. Is it really a reaction between the poles or between the magnetic fields which they represent? One may get a point of view in regard to this by considering the reaction between the magnetic field surrounding a straight conductor and a uniform magnetic field at right angles to it. In Fig. 96 is shown the cross-section of a conductor in which an electric current is flowing away from the reader. There are no poles to this field. The field H is uniform. The poles, therefore, are, for all practical purposes, at infinity. Nevertheless, these two fields react on each other in a direction normal both to the conductor and to the field H. This is shown in Fig. 97 where the resultant of the two fields of Fig. 96 is portrayed. An experimental demonstration of this force may be made as indicated in Fig. 98. The mechanical urge on the conductor is toward the weaker part of the resultant field. It is as though the resultant magnetic lines of force tried to straighten out, shorten, and become parallel. In doing so they carry the conductor along with them. If this way of picturing magnetic fields is permissible, then this tendency of magnetic lines to straighten, shorten, and become parallel seems to be a general point of view and accounts for the forces between all magnetic fields.

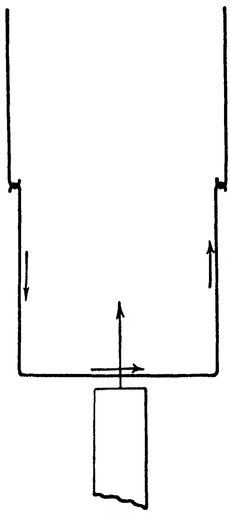


Fig. 98.—An experimental demonstration of the reaction between the magnetic field surrounding a conductor and the magnetic field in which it is placed. The force on the horizontal conductor is toward the reader.

For a great many years our textbooks of physics have usually had a page or more dealing with so-called *unipolar motors* and *induction.* This problem is again one of reactions between magnetic fields. Unfortunately the physicists have explained these motors as though magnetic poles could be isolated. Inasmuch as a *magnetic pole cannot be isolated* like an electric charge can, it is obvious that the concept is fundamentally wrong. As a result, many errors have crept into our literature on this subject, which has been called unipolar motors and unipolar inductors.

In speaking of an *isolated electric charge* we think of the electric lines of force radiating from it, uniformly, in all directions (Fig. 99). In trying to draw an analogy between electric charges and magnetic poles we have tried to think of an isolated magnetic pole in the same way. No matter how far one magnetic pole is from its companion pole there will be just as many lines of force running to it as away from it (Fig. 100). In all of our thinking it would be far better to think in terms of *di-poles* instead of *uni-poles.*

In considering unipolar motors it will be a help if we start out with Oersted's discovery (Sec. **11**). If C (Fig. 101) is a

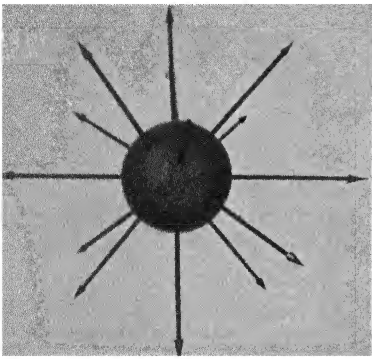


Fig. 99.—The electric lines of force radiate uniformly in all directions from an isolated electric charge.

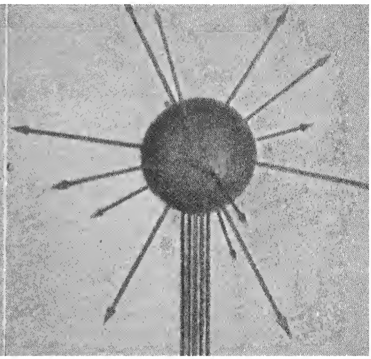


Fig. 100.—In the case of a single magnetic pole as many lines of force run to it as away from it.

closed circuit in which an electric current is flowing, magnetic lines of force will encircle the conductor in closed paths. This makes the magnetic lines of force link with the circuit C like two links of a simple chain.

In 1834 Faraday discovered that if the magnetic lines of force linked with a closed conductor varied, an electric current would be set up in the circuit. Electrical engineers have consistently used this idea of the linkage of magnetic lines of force with electric circuits and, as a result, have not fallen into the errors that others have who have talked about unipoles (isolated poles).

In the case of Oersted's discovery, it was found that the field at any point outside the conductor was:

$$H = \frac{2I}{R}, \tag{21}$$

i.e., it varies inversely as the distance from the center of the conductor. Maxwell[1] has given an experimental proof of this law and has also helped in the misleading idea of unipolar rotation. Figure 102 shows the scheme devised by Maxwell. On a small platform, concentric with the conductor and suspended by a fiber, two similar bar magnets are laid with like poles end-on to the axis of rotation. Maxwell thought of the two N poles tending to go in one direction, and the two S poles going in the opposite direction. The two inner poles exert a counterclockwise moment of turning,

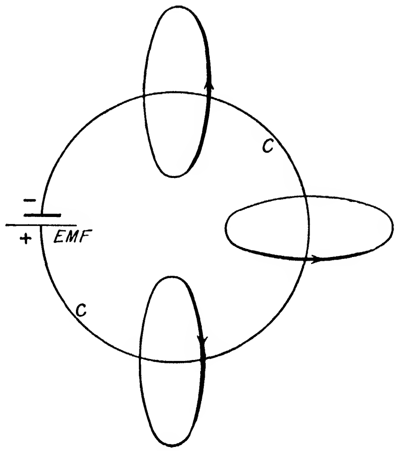


FIG. 101.—A conductor with an electric current in it has magnetic lines of force set up about it. Oersted's discovery. The magnetic lines of force are linked with the circuit.

$$M_1 = m_1H_1R_1 + m_2H_1R_1, \tag{161}$$

while the two outer, or S poles, have a corresponding clockwise moment of turning,

$$M_2 = m_1H_2R_2 + m_2H_2R_2. \tag{162}$$

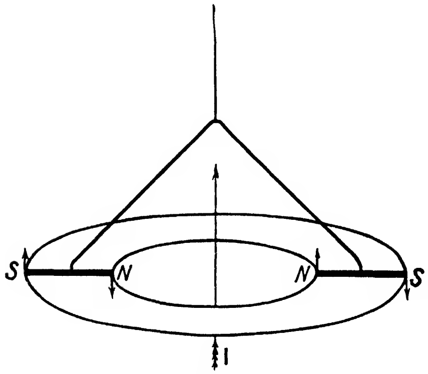


FIG. 102.—Maxwell's method to demonstrate the inverse law of the magnetic field surrounding an electric conductor.

If one moment is greater than the other, rotation will occur in the direction of greatest moment. Maxwell found, however, that no rotation occurred and, therefore, since $M_1 = M_2$,

$$(m_1 + m_2)R_1H_1 = (m_1 + m_2)R_2H_2, \tag{163}$$

or

$$\frac{H_1}{H_2} = \frac{R_2}{R_1}.$$

[1] MAXWELL, "Electricity and Magnetism," 2d ed., vol. II, p. 130; EBERT, "Magnetische Kraftfelder," p. 166, 1897; NORTHRUP, *Phys. Rev.*, **24**, 474, 1907.

This states that the magnetic force varies inversely as the distance from the center of the conductor. The same argument follows if one of the two magnets has its axis turned through 180°. Then,

$$M_1 = m_1H_1R_1 + m_2H_2R_2,$$

and

$$M_2 = m_1H_2R_2 + m_2H_1R_1$$

$$H_1R_1(m_1 - m_2) = H_2R_2(m_1 - m_2), \text{ when } M_1 = M_2.$$

$$\frac{H_1}{H_2} = \frac{R_2}{R_1}. \tag{164}$$

A far better experimental demonstration was given by Biot and Savart[1] who observed the rate of swing of a small magnetometer when situated at points to the magnetic east of a vertical current and also when subject to the earth's field only.

Let n_1 and n_2 be the number of swings of the needle per minute at distances d_1 and d_2 when n is the number due to the earth's field alone. Then,

$$\frac{(n_1^2 - n^2)}{(n_2^2 - n^2)} = \frac{d_2}{d_1}. \tag{165}$$

The method of treating this problem from the standpoint of isolated poles has carried over into the treatment of so-called unipolar motors and inductors.

In Fig. 103 is shown a common type of what has been designated as a unipolar motor. Figure 104 is another and somewhat simpler form. NS is a permanent magnet. Along its axis and out at the middle is carried an insulated wire. The outer end of this wire dips in an annular trough of mercury at M. The wire extending out from the magnet will be called the "side arm" of the exterior circuit. The wire which extends from the annular trough to infinity will be called the "fixed arm" of the exterior circuit.

When an electric current flows upward through the wire in the magnet and out through the side arm, a magnetic field encircles the wire all along its length. According to the old conceptions of unipolar motors the N pole should move around in this field and continue to do so as long as the electric current flows. The torque producing this rotation should be:

$$T = mHR = m\frac{2I}{R}R$$

$$= 2mI, \tag{166}$$

[1] Biot and Savart, *Ann. chim. phys.*, **15**, 222, 1820; Hadley, "Electricity and Magnetism for Students," p. 251, 1908.

where m is the pole strength of the magnet and I is the current flowing in the circuit. This equation does not indicate that any changes will occur due to changes in the exterior circuit, and yet Zeleny and Page[1] have recently shown that, as the length of the side arm increased, the torque decreased. Evidently the unipolar concept is misleading.

If we look at the magnet in Fig. 104 as a dipole, it has closed magnetic lines of force running from the N end to the S end.

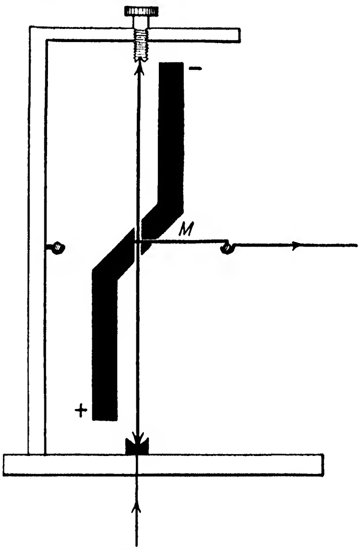


FIG. 103.—One form of a unipolar motor. The lower magnetic pole moves around the conductor.

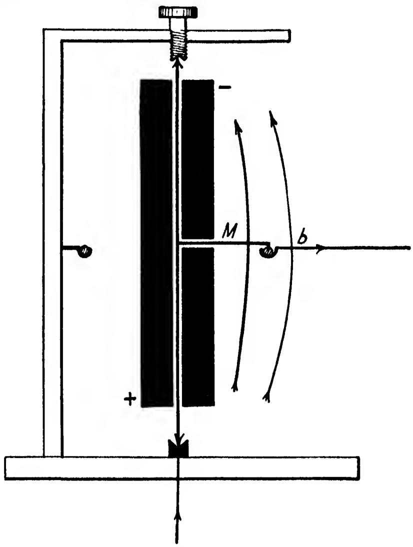


FIG. 104.—Another form of unipolar motor. These motors will act as generators also.

These lines of force run at right angles both to the side arm and the fixed arm. There will be a reaction between the magnetic field of the magnet and the magnetic field surrounding the side and fixed arms. This reaction will cause the magnetic lines of force at b to push the fixed arm forward or they, in turn, will be pushed in the opposite direction. Since the lines of force may be thought of as attached to the magnet, the magnet will rotate in a counter-clockwise direction as viewed from above. The reaction between the magnetic lines of force from the magnet and those surrounding the side arm will be in the same direction, but since both are bound to the magnet they balance each other.

[1] ZELENY and PAGE, *Phys. Rev.*, **24**, 544, 1924.

It will be noted that the direction of rotation as thus worked out is the same as that deduced by the unipolar concept. Furthermore, it brings out very clearly that, as the side arm is lengthened, the turning moment will be decreased because there is less of the fixed arm on which to react. The manufacturers of these forms of motors will do well to get the annular trough as close to the magnet as possible.

The torque which is applied to the magnet, may also be calculated from the standpoint of dealing with a dipole. Let $d\rho$ (Fig. 105) represent a very short section of the fixed arm. The magnetic fields acting at $d\rho$, due to the magnet NS, are:

$$\text{Magnetic field at } d\rho \text{ due to } N = \frac{m}{d_2^2}$$

$$\text{Magnetic field at } d\rho \text{ due to } S = \frac{m}{d_1^2}.$$

$d_1 = d_2 = \sqrt{\rho^2 + l^2}$, where $2l$ = distance between the poles of magnet.

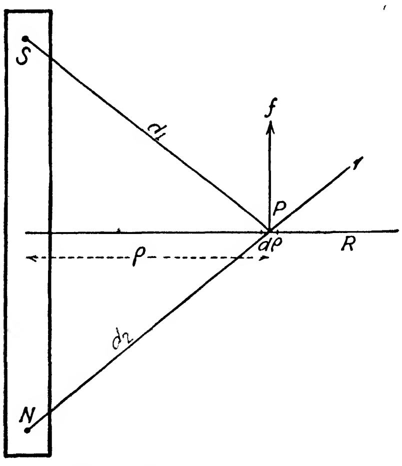


FIG. 105.—The field at $d\rho$ due to the magnet NS is f. This seeks to push the conductor R toward the reader.

Resultant magnetic field at $d\rho$ due to $NS \equiv f$ which is directed upward.

$$2l:f = d_2:\frac{m}{(\sqrt{\rho^2 + l^2})^2} \qquad (167)$$

$$2l:f = \sqrt{\rho^2 + l^2}:\frac{m}{\rho^2 + l^2}$$

$$f = \frac{2lm}{(\rho^2 + l^2)^{3/2}}.$$

If we make R the length of the fixed arm, and ρ the distance $d\rho$ is from the center of the magnet, then the magnetic field at P pushes the element $d\rho$ forward. This gives a moment of turning to the magnet,

$$dT = \frac{2Ilm\rho d\rho}{(\rho^2 + l^2)^{3/2}}. \qquad (168)$$

Integrating between the limits o and R:

$$\int_o^R dT = \int_o^R \frac{2Ilm\rho d\rho}{(\rho^2 + l^2)^{3/2}}.$$

Whence,

$$T = 2mI\left\{1 - \frac{1}{\sqrt{1 + (R/l)^2}}\right\}.$$

If R, the length of the fixed arm, becomes zero, *i.e.*, the side arm is infinitely long, then $T = 0$ which is in harmony with what Zeleny and Page found.

If $R = \infty$, then

$$T = 2mI, \tag{166}$$

which is the value derived from the unipolar standpoint. There are times when the concept of an isolated magnetic pole is useful, but it must be used with discretion.

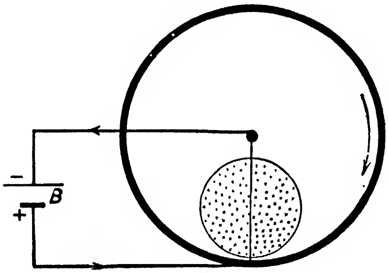


FIG. 106.—Barlow's wheel. The positive pole of the battery is connected to the periphery of the disk by a sliding contact. The negative pole is connected by a similar contact with the axle of the disk.

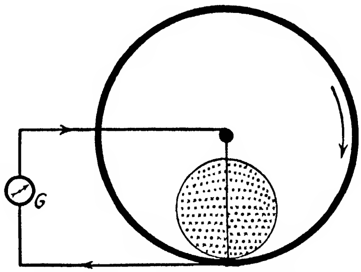


FIG. 107.—Faraday's disk. Mechanically constructed the same as Barlow's wheel. Barlow's wheel is a motor while Faraday's disk is a generator.

Our textbooks have used this idea of isolated magnetic poles so long that we cannot easily break away from such a conception. Whenever, in this book, a remark is made about the effect of a magnetic pole it must be held clearly in mind that it is done conventionally and with many reservations. A perusal of the papers which have been written on this subject will clearly indicate the pitfalls which have been devised by those adhering to the unipolar concept.

Another feature of some of our textbooks, which confuses the minds of many students, is placing emphasis on Barlow's wheel and Faraday's disk, as though they were some special experiment. Some of the so-called unipolar motors are special forms of Barlow's disk. Whereas in the former the magnets rotate, in the latter the magnets are fixed and the conductors rotate. Faraday's rotation apparatus[1] is a special form of

[1] See Bibliography.

Barlow's wheel. Faraday's disk is the reciprocal experiment of Barlow's wheel. Faraday's disk is a dipole generator while Barlow's wheel is a dipole motor. Figures 106 and 107 show analogous experiments (diagrammatically), for Barlow's wheel and Faraday's disk. In both cases a copper disk is mounted on an axle, which rotates with the plane of the disk normal to a magnetic field. If a current is sent from the axle to the periphery of the wheel, the disk acts as a motor. If the wheel rotates mechanically in the magnetic field an electromotive force is set up between the axle and the periphery of the disk. All of these various devices are reversible machines. If one acts as a motor it will also function as a dynamo.

There are many forms and variations given to these rotating and generating devices, but all are readily explained in terms of dipoles.

It would form a most interesting collection to bring together the models of all the different motors and generators which have been devised in the past as unipolar machines.

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CHAPTER III

MAGNETO-ACOUSTICS

41. Production of Sound by Magnetization.—All types of sound have as their source some mechanically vibrating body. From the caption, therefore, it is obvious that this chapter has to do with any phenomenon wherein the vibrations of a body are produced or affected by a magnetic field and, conversely, the effects of vibrations on magnetic properties.

One of the earliest to publish his observations on this subject was Page.[1] He found that a coil of 40 turns, placed vertically between the poles of a horse-shoe magnet, produced a distinct tone in the magnet when an electric current was started or stopped in the coil. Similarly, it has been observed that when an electromagnet was rotated between the poles of a horse-shoe magnet a distinct tone was heard as from a tuning-fork. In both cases it would appear that the momentary field attracted or repelled the poles of the magnet and, as it were, struck them a blow and set them into vibration. The tone emitted by a tuning-fork thus struck would be that of its own free period. If, on the other hand, an alternating current was sent through the coil in Page's experiment it would be a case of forced vibrations wherein the tone would be that of the frequency of the imposed force. This would be illustrated by the action of the tympanum in the telephone receiver or the armature of the telegraph sounder.

Instead of these transverse vibrations Marrian[2] found that rods placed axially in coils or solenoids could be made to set up longitudinal vibrations in a perfectly analogous way; *i.e.*, if the current were started or stopped the rod gave a distinct tone which was the fundamental for its longitudinal vibrations. If an alternating current is sent through the coil it will respond in frequency to that of the imposed force. Matteucci[3] studied the effect of tension on the tone of the longitudinal vibration and observed no other effect than that the intensity was increased

[1] PAGE, *Poggendorff Ann.*, **43**, 411, 1838; **63**, 530, 1844.

[2] MARRIAN, *Philos. Mag.*, **25**, 382, 1844.

[3] MATTEUCCI, *Arch. d. Sci.*, **5**, 389, 1845.

when stretched. Wertheim,[1] studying the same phenomenon, found the tone independent of cross-section of rods.

In Sec. **38** the effect of a change in length due to a magnetic field was discussed. In the phenomena, studied by Marrian, Matteucci, and Wertheim, the change in length occurred when the current was applied or broken, and this change in length gave rise to the longitudinal vibrations. Concomitant with the production of the longitudinal vibrations is the sound known as the "magnetic tick." When the field is applied a very sudden change in length occurs, and it is this sudden change in length which makes the rod emit a click as though it has been struck with a small hammer.

Honda and Shimizu[2] studied the effect of various frequencies on the rods set into longitudinal vibrations and increased the frequency up to 150. Recent investigators have run this periodicity up much higher and have pieces of ferromagnetic metals in place of quartz oscillators for obtaining high-frequency vibrations.[3] In the paper by Honda and Shimizu, just mentioned, they discuss a number of papers bearing on the problem of sound produced by magnetic fields.

42. Modification of Sounds by Magnetic Fields.—A study of electrically driven tuning-forks had shown that the electric contacts[4] and the magnetic fields[5] of the driving electromagnets influenced the amplitude of vibration of the forks. In the case of the magnetism of the electromagnets, it was found that the amplitude was inversely proportional to the distance the poles were from the prongs of the fork. Effects of this sort led others[6] to investigate the effect of a magnetic field on the frequency of a tuning-fork. Kirstein[7] has also made a very thorough investigation and found among other effects the following:

1. That if a tuning-fork vibrates in a magnetic field, so that the lines of force are normal to the plane of vibration, the frequency is increased and decreased when the plane is parallel.

2. The change in the frequency is approximately proportional to the field strength.

[1] Wertheim, *Poggendorff Ann.*, **77**, 43, 1849.

[2] Honda and Shimizu, *Philos. Mag.*, **4**, 645, 1902.

[3] Pierce, *Proc.* Amer. Acad. Arts Sci., **63**, 1, 1928.

[4] Loewenherz, *Zeitsch. für Instrumentenk.*, **8**, 350, 1880.

[5] Mercadier, *Compt. rend.*, **76**, 431, 1873.

[6] Maurain, *Compt. rend.*, **121**, 248, 1895.

[7] Kirstein, *Physikal. Zeitsch.*, **4**, 829, 1903.

3. The effect of the magnetism on the tuning-fork is only a temporary one.

4. The decrease in frequency noted in 1 is greater than the increase for the same field strength.

5. When the plane of vibration of the fork and the lines of force make an angle of 45° with each other no change in frequency occurs.

Undoubtedly some of these effects are related to some of the magnetostrictive effects which have been discussed in Chap. II. The small change in length of the prongs of a tuning-fork when magnetized must make a slight change in the frequency. The change in elastic constants must also contribute its effects. There are many points which need more detailed investigation. Not only is there the effect of the magnetic field on the frequency of the tuning-fork but, in an analogous fashion, the frequency of the longitudinal vibrations of a rod must also be changed when magnetized. The effects of a magnetic field on longitudinal vibrations could be studied by means of Lissajou's figures.

43. Effect of Sound Vibrations on Magnetism.—This is the reciprocal effect of that discussed in the preceding section. It has already been mentioned in the section describing the processes of demagnetization. This effect is very pronounced in the case of nickel. If a nickel bar 10 cm. long and 2 mm. in diameter is magnetized and then suspended on a phosphor-bronze fiber it will vibrate in a magnetic field with a frequency given by the expression,

$$\frac{1}{n} = 2\pi\sqrt{\frac{K}{MH}}. \tag{169}$$

If the nickel rod is removed from its suspension and stroked, either longitudinally or circularly, longitudinal or circular vibrations will be set up in the rod, causing a decrease in the magnetic moment and therefore a decrease in the frequency of vibrations. No matter whether these vibrations are set up mechanically or by means of sound waves, the same results occur, and one ought to get interesting results from the use of Wood's high-frequency sound waves.

Not only do vibrations decrease the magnetization of a body already magnetized, but bodies in the process of being mag-

netized require a smaller magnetomotive force to attain a certain intensity if vibrations are applied than when undisturbed.[1]

Warburg[2] has contributed an interesting experiment bearing upon this general topic. He magnetized a long iron wire and set it into longitudinal vibrations. An electrodynamometer was connected to a coil surrounding a portion of the wire and, when the wire was vibrating longitudinally, the electrodynamometer gave a deflection indicating that there was a periodic variation in the intensity of magnetization. The compressions and rarefactions of the wire affected the intensity of magnetization as in the Villari effect, and so induced currents were set up in the coil and the electrodynamometer. This experiment should be extended to other materials, especially to nickel. In iron there should be some point corresponding to a Villari reversal point, if the Villari effect is the explanation of Warburg's experiment.

44. Barkhausen Effect.—One of the recent discoveries of a sound originating from a magnetic field is that which was first observed by Barkhausen.[3] If a coil of about 50,000 turns of No. 24 double-cotton-covered wire is wound about a strip of thin sheet iron, and the terminals of the coil attached to the input of an amplifier, a very feeble electromotive force will be developed in the coil as the sheet iron is magnetized. As a result of these very small, induced currents one hears in the loud speaker attached to the amplifier a murmuring or sighing like the escaping of air or steam under pressure. This is explained by saying that the process of magnetization is not a continuous operation. As the magnetizing force is increased, the elementary magnets of the iron flop over in varying-sized groups and, in thus turning, produce an electromotive force in the coil surrounding the iron (Fig. 108). These small inductive currents come with irregular frequency and magnitude, and thus give rise to a murmuring sound in the loud speaker. This is not in reality a noise due to magnetization any more than there is a sound of speech carried along a telephone wire. Could one observe an ordinary induction curve in great detail it would not be smooth, as the one shown in Fig. 46, but would be very irregular as Forrer[4] has

[1] BECKNELL, *Phys. Rev.*, **8**, 504, 1916.
[2] WARBURG, *Poggendorff Ann.*, **139**, 499, 1870.
[3] BARKHAUSEN, *Physikal. Zeitsch.*, **20**, 401, 1919.
[4] FORRER, *Jour. phys. et le radium*, **7**, 109, 1926.

shown to be the case for nickel and Van der Pol[1] for nickel-steel and iron.

Among others who have investigated the Barkhausen effect may be mentioned Gerlach and Lertes, Zschiesche, Williams, Tyndall, Pfaffenberger and Bozorth.[2] This effect discovered by Barkhausen seems to be intimately connected with the Joule

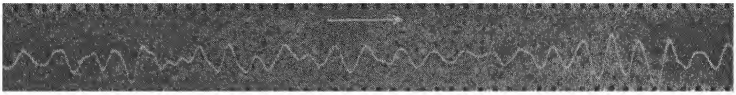

FIG. 108.—Oscillogram of Barkhausen effect. Arrow indicates direction that film moved. Distance from one perforation to the next indicates 1/509 second.

magnetostrictive effect as Zschiesche has shown. It has also been found that the magnetic field, at which abrupt changes in the elongation or contraction of a single crystal of silicon steel occur, is also the point at which the Barkhausen noise swells out loudest. Among recent discoveries this one of Barkhausen has been very illuminating concerning the mechanism of magnetization.

[1] VAN DER POL, *Proc. Acad.*, Amsterdam, **23**, 637, 1921; **23**, 980, 1922.

[2] GERLACH and LERTES, *Zeitsch. für Phys.*, **4**, 383, 1921; *Physikal. Zeitsch.*, **22**, 568, 1921;

ZSCHIESCHE, *Zeitsch. für Phys.*, **11**, 201, 1923;

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CHAPTER IV

MAGNETO-ELECTRICS

45. Development of Electromotive Force by Varying Magnetic Fields.—In Sec. **40** it was pointed out that if the number of magnetic lines of force linked with an electric circuit were varied, there would be set up in the conductor an electromotive force, and that such an effect was the reciprocal of the discovery by Oersted. Field intensity is a vector quantity. Any variation in a magnetic field, either in direction or in magnitude, gives rise to an induced electromotive force. Conversely, any change in the vector quantity of current strength produces a magnetomotive force.

It would take us too far from the main path to enter upon any discussion of the fields to which the discoveries of Oersted and Faraday have led us in recent years, in our electric light and power developments. The development of magnetic fields and the materials whereby high permanent fields may be maintained, or high variable fields may be produced, with a minimum of core loss, rest upon a thorough study of the processes which occur when a body is magnetized. No one has any conception of the value it might be to industry and civilization were we to have a clearer idea of these fundamental phenomena.

Why is it that when a magnetic field is varying near a conductor, either in direction or magnitude, it seizes the electrons in that conductor and piles up an electromotive force? We have no idea of the machinery whereby this is accomplished.

This interrelation between electrons in motion and magnetic fields, and the tendency for varying magnetic fields to set electrons in motion, forms one of the most fascinating problems we have in magnetic phenomena. This problem lies at the very heart of magnetism. The processes go on, all about us. The transformers just outside our houses which step the high voltage of the line down to a safe voltage, to use for lighting purposes in our homes, are a common example, and yet how little we know about it all! A most beautiful and striking example of induction

effects is in the electrodeless discharge,[1] in which the varying magnetic field of a coil surrounding an exhausted glass bulb takes hold of the electric charges inside of the bulb and sets them in motion in curves concentric with the coils. Our present beliefs lead us to the idea that no magnetic field exists without the movement of *electrons*. While we talk about *magnetons*, they can never be so fundamental a conception as the electron if our point of view is correct.

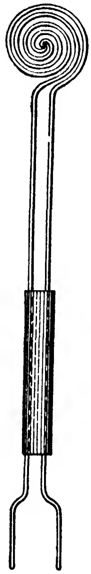

FIG. 109.—A flat bismuth spiral for measuring strong magnetic fields.

This chapter has to do primarily with the effects of a magnetic field upon electrons at rest and in motion. While it presents quite a number of cases where, it seems, we may look in on the processes which are going on when a magnetic field reacts upon the electrons, yet we are still very much in the dark concerning the fundamental problem. The *magnitude* of the electromotive force developed, due to a varying magnetic field, was given in Eq. (1). The *direction* of the electromotive force thus produced, is given by Lenz's law which says that the direction of the induced current is always in such a direction as to set up a magnetic field opposing the inducing field.

46. Change in Resistance Due to a Magnetic Field.—Whether a substance is a ferromagnetic body or not, it has been found that a conductor has its apparent resistance changed by a magnetic field. This effect was first observed by Sir William Thomson[2] in 1856 who found that the resistance of iron was increased when magnetized *longitudinally*. Later he found that the same relation held for nickel. The outstanding example of this phenomenon is the bismuth spiral, which is frequently used for measuring magnetic field strengths. Bismuth is a diamagnetic substance and yet shows the greatest change in resistance, due to a magnetic field, of any substance we know at present. A fine wire of bismuth is formed into a spiral like that shown in Fig. 109 and placed with the plane of the coil at right angles to the field. The resistance is then measured for various field strengths which are known and a curve plotted for the same. With this calibration

[1] DAVIS, *Phys. Rev.*, **20**, 129, 1905.

[2] THOMSON, *Philos. Trans.*, **146**, 736, 1856; *Proc. Roy. Soc.*, **8**, 546, 1857.

curve, an example of which is shown in Fig. 110, any unknown field may be measured. The difference between the resistance with and without the field H, divided by the resistance with zero field, is plotted as ordinates. All measurements with the bismuth spiral should be made, as far as possible, at the same temperature as the calibration temperature. It is advisable, therefore, to keep the current in the spiral as low as possible while determining its resistance. It will also help if the current is shut off while the coil is actually not in use. If the temperature at which the spiral is used does vary from that at which it was calibrated, correction for the temperature coefficient of bismuth should be

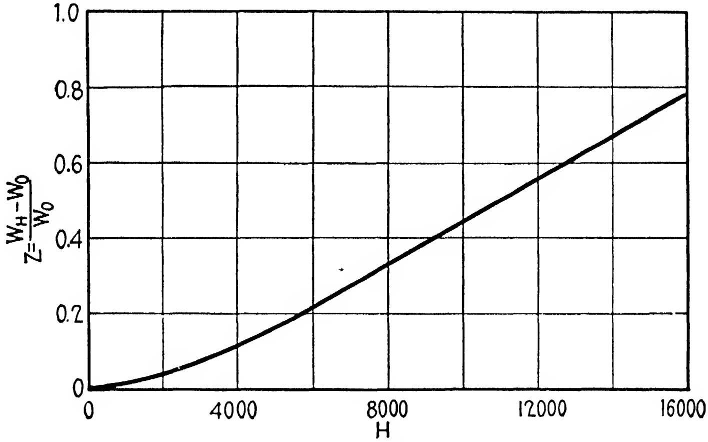

FIG. 110.—A calibration curve for a bismuth spiral.

applied. This method is intended primarily for measurement of intense magnetic fields (1,000 gausses or more).

The investigation of change in resistance in various conductors has been undertaken by numerous workers. Perhaps, especial interest is to be attached to the work with ferromagnetic conductors in which there seems to be some relation between this effect and the Joule magnetostrictive effects.

Observers seem to be pretty well agreed as to the character of the change of resistance, *i.e.*, that in iron and nickel there is an increase in resistance when the magnetic field parallels the conductor, and a decrease in resistance when the field is normal. Bismuth and antimony show an increase under all conditions. Substances which are practically indifferent to a magnetizing force show little or no change in resistance due to a magnetic

field. As regards the relation between the change in resistance and the intensity of magnetization or the magnetizing force, there are the most contradictory results. Doubtless, the discrepancies between the various results obtained are due to impurities in the materials investigated. Very small quantities of impurities will vary the change in resistance tremendously.

Not only are magnetic and non-magnetic conductors to be differentiated between when studying the changes in resistance

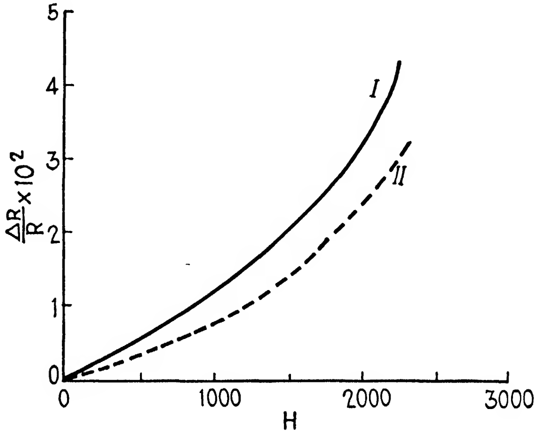

FIG. 111.—Curve I shows the change in resistance of bismuth when the magnetic field is parallel to the conductor. In curve II the field and the bismuth conductor are at right angles to each other.

due to a magnetic field, but a difference arises when the conductor is placed so that the electrons move either normally or parallel to the magnetizing force. Barlow[1] studied the non-magnetic metal, bismuth, in both longitudinal and transverse fields. Figure 111 shows the manner in which the resistance increased in both cases. Similarly, Jones and Molam[2] did an excellent piece of work in studying the change in resistance of nickel (magnetic) as the conductor was turned from a normal to a longitudinal position in the magnetic field.

The work of Jewett[3] and of Richtmyer and Curtis[4] on sputtered bismuth films leads to the conclusion that the crystalline structure of the conductor plays an exceedingly important rôle in

[1] BARLOW, *Proc. Roy. Soc.*, **71**, 30, 1902; *Ann. der Phys.*, **12**, 897, 1903.

[2] JONES and MOLAM, *Philos. Mag.*, **27**, 649, 1914.

[3] JEWETT, *Phys. Rev.*, **16**, 51, 1903.

[4] RICHTMYER and CURTIS, *Phys. Rev.*, **15**, 465, 1920.

all of these phenomena. Kaya[1] has brought out the same point in a study of the magneto-resistance effect in a single crystal of nickel.

> In the case of the longitudinal effect, every direction of the axes shows increase of resistance, the amounts of which are in the decreasing order of 111, 110, and 100. In the case of the transverse effect, both increase and decrease of resistance are observable according to the directions of the magnetic field.

Temperature, hydrostatic pressure, and alternating currents are some of the other factors which vary the resistance of a conductor in a magnetic field.

Changes in resistance due to a magnetic field have been found in liquid metals,[2] electrolytes,[3] gases,[4] and in flames.[5] Wherever the motion of charged particles exists, there a magnetic field will influence them.

Campbell[6] sums it up as follows: (1) Magneto-resistance of Non-magnetic Metals—

> In general, the magneto-resistance of *non-magnetic metals* in both transverse and longitudinal fields is an *increase.* The rate of increase is greater in higher fields and at lower temperatures. The change in resistance is proportional to the square of the field strength, and depends in magnitude upon the direction with respect to the crystallographic axis.

(2) Magneto-resistance of Ferromagnetic Metals—

> The *ferromagnetic* metals, in general, *increase* in resistance in a *longitudinal* field, rapidly at first, followed by an approach to a limiting value at magnetic saturation. In a transverse field the resistance of these metals decreases, slowly at first, then rapidly, followed by an approach to a saturation point.

At the present time Kapitza[7] is carrying out in the Cavendish Laboratory a very important study of the change in resistance due to a magnetic field. In the first place, he is using what is, in all probability, the best collection of pure elements that

[1] KAYA, *Sci. Repts. Tôhoku Imp. Univ.*, **17**, 1027, 1928.

[2] ROSSI, *Nuov. Cim.*, **2**, 337, 1911.

[3] SVEDBURG, *Ann. der Phys.*, **44**, 1121, 1914.

[4] IVES, *Phys. Rev.*, **12**, 293, 1918.

[5] WILSON, *Proc. Roy. Soc.*, **82**, 595, 1909.

[6] CAMPBELL, "Galvanomagnetic and Thermomagnetic Effects," pp. 158–210, 1923.

[7] KAPITZA, *Proc. Roy. Soc.*, *A*, **119**, 358, 1928.

has been assembled for any such purpose. In the second place, he is able to carry on his experiments with field strengths as high as 350,000 gausses. His final results, when published, will create a new chapter in this particular field.

Historically, the change in resistance due to a magnetic field was the first of a series of magneto-electric phenomena to be discovered. In his initial papers on the Dynamical Theory of Heat, Sir William Thomson predicted the possibility of some of these effects, being led to this point of view by the fact that magnetized iron showed different thermo-electric properties from the unmagnetized. Several looked for a possible effect of a magnetic field upon an electric current, but failed to find it. Not until 1879 did Hall[1] succeed in finding a "New Action of the Magnet on Electric Currents."

47. Galvanomagnetic Effects.—The discovery by Thomson, discussed in the preceding section, stimulated a large amount of research on the effect of a magnetic field on the flow of electricity. At the same time there was found to be an analogous effect on the flow of heat when the conductor was subjected to a magnetic field. Eight specific effects were discovered. Four, which have to do with the effect of a magnetic field on the flow of electricity, are called galvanomagnetic effects, and the other four, which deal with the flow of heat, are called thermomagnetic effects. The last four will be discussed in the next chapter. Figure 112 illustrates schematically the eight different effects and are brought together at this point to show their analogy and interrelation.

a. Hall Effect.—Hall[2] discovered that, when an electric current flows through a thin plate of metal, a magnetic field would rotate the equipotential lines when the field is normal to the lines of flow. This may be visualized in Fig. 113. The electric current enters one end of the rectangular plate at point A and leaves at point B. The magnetic field is directed into the page in all the diagrams in Fig. 112. The lines of flow for the Hall effect will be symmetrically distributed as shown in Fig. 113, and the points C and D will be equipotential points. On applying a field at right angles to this plate and arrangement of lines of flow, there ensues a redistribution of the equipotential lines, as shown in Fig. 114. The equipotential lines, which are always normal to the lines of

[1] Hall, *Philos. Mag.*, **9**, 225, 1880.

[2] Hall, *Amer. Jour. Math.*, **2**, 287, 1879; *Philos. Mag.*, **9**, 225, 1880.

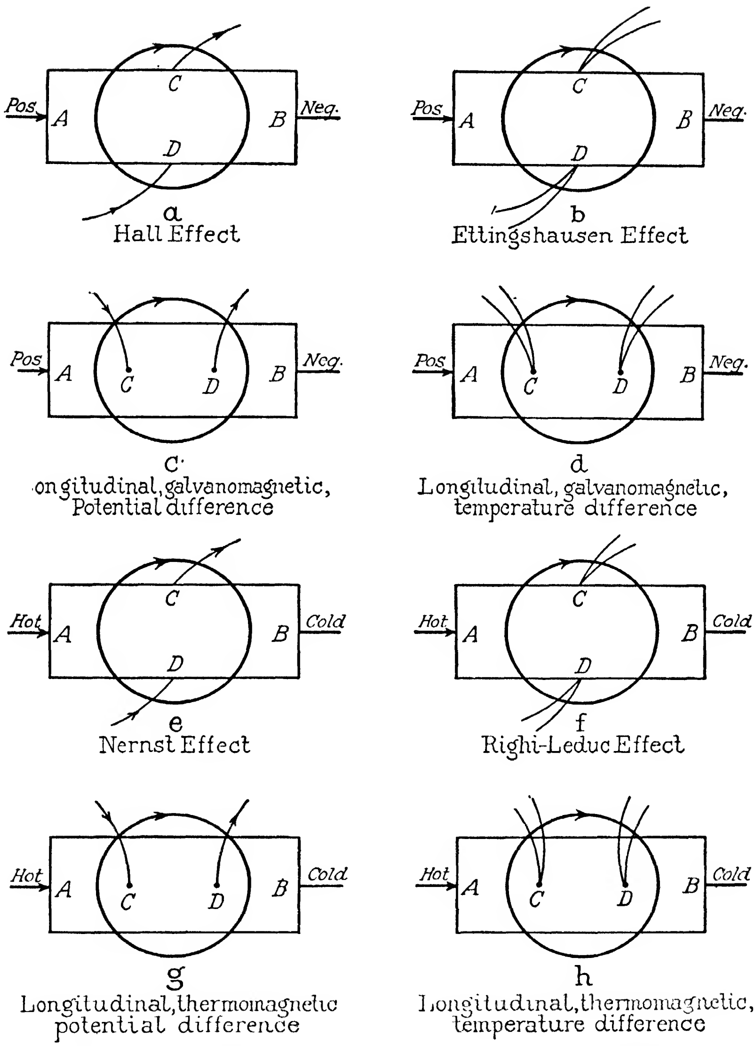

FIG. 112.—A comparison of galvanomagnetic and thermomagnetic effects. In the galvanomagnetic effects the current enters the plate at a point, while in the thermomagnetic effects the heat enters uniformly over the entire end of the plate.

flow, will be rotated so that C and D are no longer equipotential. When they are connected to a galvanometer, a deflection occurs. The removal of the magnetic field gives zero deflection once more. In Fig. 114 the equipotential lines have been rotated clockwise, which effect is called the positive Hall effect. Some metals reverse the direction of rotation, and then a negative Hall coefficient is obtained. It may be inferred that the reversal of a magnetic field or direction of flow of the current also reverses the direction of rotation. This effect, discovered by Hall, varies with current density, magnetic field, temperature, and direction of crystallographic axes. A prodigious amount of work has been done along these lines and an interesting résumé may be found in Campbell's book.[1]

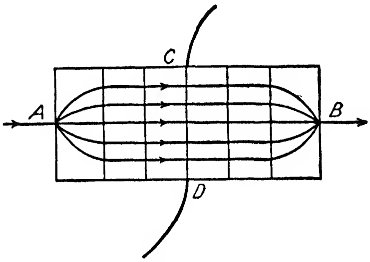

FIG. 113.—Lines of electric flow in a conducting sheet.

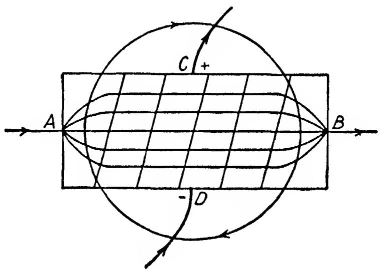

FIG. 114.—In a magnetic field the equipotential lines are rotated, but the stream lines of the primary current are not bent.

b. Ettingshausen Effect.—In Fig. 112*b* an electric current flows through the plate as in the Hall effect. Heat effects will be produced as in any conductor. Referring to Fig. 113 the lines of electric flow will be the same, and C and D will have the same temperature. On applying the magnetic field perpendicularly to the plate, the equipotential lines of flow will be rotated as before but no longer will C and D be at equal temperatures. This may be indicated by attaching the junctions of a thermocouple at points C and D. Without the magnetic field there is no deflection of the galvanometer, showing C and D are at the same temperature, but exciting the magnetic field shows that the plate has "suffered an unequal change in temperature at its lateral edges." This is known as the Ettingshausen[2]

[1] CAMPBELL, "Galvanomagnetic and Thermomagnetic Effects," 1923.
[2] ETTINGSHAUSEN, *Wiedemann Ann.*, **31**, 737, 1887; **33**, 126, 1888.

effect. In Fig. 115 is shown the conventional positive Ettingshausen effect for bismuth and in Fig. 116 the negative effect as found in iron.

c. Galvanomagnetic, Longitudinal, Potential Difference.—If a metallic plate is arranged in a magnetic field as for the two preceding effects, it will be found that a change in the potential difference between the points C and D (Fig. 112*c*) is created when the magnetic field is applied. By Ohm's law this means that if the current through the plate is maintained constant, the change in electromotive force between C and D is due to a change in resistance. In other words, the galvanomagnetic, longitudinal,

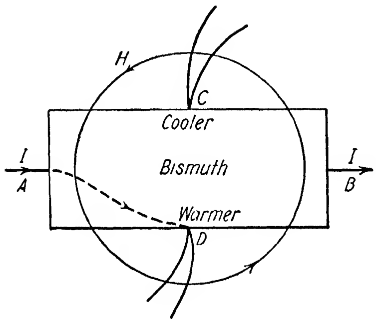

FIG. 115.—In the Ettingshausen effect if the extra-current flows in the same direction as the magnetizing current it is called the positive effect. Bismuth is an example.

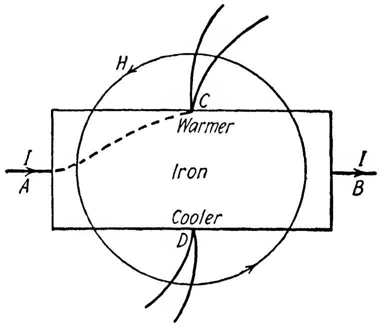

FIG. 116.—In the Ettingshausen effect if the flow of heat is from the entrance end I, and opposed to the current of the magnetizing field, the effect is said to be negative. The iron plate indicates this relation.

potential difference is the effect discovered by Thomson and discussed in Sec. **46** for which the magnetic field is normal to the lines of flow. As has been indicated already, this was the earliest of these effects to be discovered. Whether this effect is called a change in resistance due to a magnetic field or the galvanomagnetic, longitudinal, potential difference, depends upon whether resistance or potential difference is being emphasized.

d. Galvanomagnetic, Longitudinal, Temperature Difference.—This effect was discovered by Nernst[1] in 1887. By analogy its name would indicate that a change in the temperature gradient between the points C and D (Fig. 112*d*) was produced when a magnetic field was applied normally to the lines of electric flow. This is a problem which needs additional work done on it. The first step would be to increase the sensitivity of the methods

[1] NERNST, *Wiedemann Ann.*, **31**, 760, 1887.

employed. Since all of these galvanomagnetic effects arise from the reaction of a magnetic field on the magnetic fields of moving electrons there are many interrelations which are very important for their bearings on the electron theory of matter. It is these interrelations[1] which are significant. Any investigation leading to further correlations will be of value.

48. Electromotive Force Due to Magnetization.—In order to set up an ordinary voltaic cell it is necessary to have two different elements dipping into an electrolyte. Must the two electrodes be different *chemically?* This suggests the possibility of the generation of an electromotive force, if the two poles of the cell are different *physically.* This has been confirmed in many different ways. Two pieces of the same metal, one strained and the other unstrained, give a small difference of potential due to inequality in physical properties. It would be an easy inference to assume that if a cell were made from magnetized and unmagnetized specimens of the same material there would be a difference of potential between them when dipped in an electrolyte. This is exactly what Gross[2] discovered in 1886. Many others[3] have investigated this effect and confirmed Gross' results. The direction of the current produced by such a cell depends upon the metal. The effect is not limited to the ferromagnetic substances but is found as well in bodies like bismuth. In general, it may be said that the unmagnetized electrode of nickel and iron is always negative. In the case of bismuth the unmagnetized pole is the positive one. Some results of Paillot for magnetized and unmagnetized elements of iron indicate a maximum effect amounting to $\frac{1}{22}$ of a volt for a field strength of 30,000 gausses. The effect is much smaller in bismuth as one might expect. Bucherer has tried to explain these changes in electromotive force as due to secondary effects. It is a problem that needs much study, for there is as yet no real explanation of the effect.

49. Change in Electromotive Force of a Voltaic Cell Due to a Magnetic Field.—Some investigators have found a change in the electromotive force of a cell when magnetized. In general, this change seems to be a decrease. Wyss[4] seems to have done the

[1] Bridgman, *Phys. Rev.*, **24,** 644, 1924.

[2] Gross, *Wiener Berichte,* **92,** 1378, 1886.

[3] Rowland, and Bell, *Philos. Mag.*, **26,** 105, 1888;
Bucherer, *Wiedemann Ann.*, **61,** 807, 1897;
Paillot, *Compt. rend.*, **131,** 1194, 1900; **132,** 1318, 1901.

[4] Wyss, "Inaugural Dissertation," Zürich, 1900.

most on this problem and has attempted to explain it on the grounds of changes in the concentration of the electrolyte. The combination which has usually been used is iron with some other metal dipped either in a ferrous or ferric-salt solution. The change in electromotive force has, in some cases, amounted to as much as $\frac{1}{20}$ of the original electromotive force, being largest in a ferric-salt solution. This latter fact favors Wyss' theory. Again, since iron was used as one electrode, one might suspect that the effect discussed in the preceding section played at least a minor rôle. If many of these obscure points could be cleared up, it would be exceedingly helpful to a better understanding of a comprehensive theory of magnetism.

50. The Thermal Electromotive Force between Magnetized and Unmagnetized Portions of the Same Substance.—That a chemical difference between two substances is not essential for a voltaic cell has further support in the phenomenon that between a magnetized and unmagnetized section of the same piece of wire a thermal electromotive force may be produced. In other words, the magnetization of a substance changes its position slightly in the scale of its thermo-electric power. This effect was observed by Sir William Thomson[1] in 1856. In the case of a couple made of magnetized and unmagnetized iron wire, the current was from the magnetized to the unmagnetized portion in the cold junction, when the wire was parallel to the field. Transverse magnetization reversed the direction of the current. A longitudinal magnetic field caused nickel to behave in a manner just opposite to that of iron. Apparently no one has studied nickel in a transverse field nor have observations been extended to other substances. This effect has its analogue in the voltaic cell produced by using magnetized and unmagnetized electrodes.

Another variation of this effect is to be found in an experiment suggested by Kelvin.[2] A continuous piece of ferromagnetic wire is bent in right angles at H and C (Fig. 117). The wire thus formed is placed symmetrically between the poles of an electromagnet. If H and C are kept hot and cold respectively, a magnetic field causes a thermal electromotive force to be developed in the wire. For weak magnetic fields the direction of the current is opposite to that for stronger fields in the case of soft iron wire (Armco).

[1] Thomson, *Philos. Trans.*, **146**, 709, 1856.

[2] Kelvin, "Mathematical and Physical Papers," vol. II, sects. 88 and 91.

The actual direction of the current is from the transversely magnetized to the longitudinally magnetized portions, through the hot junction, when weakly magnetized.

One way of looking at this case is to say that the section *HC* is magnetized longitudinally and the portions *HG* and *CG* are magnetized transversely. Inasmuch as the transverse magnetization is not nearly so strong as the longitudinal, it may appear that it is really a thermal electromotive force between a magnetized and unmagnetized portion.

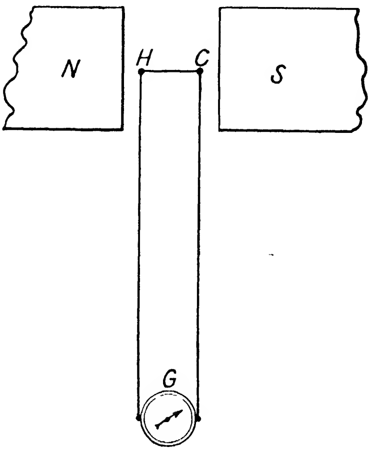

FIG. 117.—A continuous piece of iron wire is bent into right angles at *H* and *C*. If *H* and *C* are kept hot and cold respectively, an electromotive force is developed when a magnetic field is applied.

51. Influence of a Magnetic Field on the Thermo-electric Power of Metals.—Given two metals between which a thermal electromotive force exists, a difference in that electromotive force will be produced by a magnetic field. Just as in Sec. **49,** given a voltaic cell with a definite electromotive force, a magnetic field will change its electromotive force, so in this effect we have an analogous phenomenon. Strouhal and Barus[1] studied a copper-iron element in a magnetic field and got a change of about one-half of 1 per cent between the magnetized and unmagnetized condition and with a difference of about 83° C. between the two junctions. Ewing[2] found that stress varied these effects. Lownds[3] investigated crystal structure with respect to orientation in the field and obtained positive results. Bachmetjew[4] found a relation between the changes in thermal electromotive forces produced by a magnetic field and the change in length due to a magnetizing force. This amounts to making this effect a secondary one. One is impressed by these repeatedly occurring correlations and yet there is lacking continually the insight to

[1] STROUHAL and BARUS, *Wiedemann Ann.*, **14,** 54, 1881; *Bull.* U. S. Geol. Survey, 1885.

[2] EWING, *Proc. Roy. Soc.*, **40,** 246, 1886.

[3] LOWNDS, *Philos. Mag.*, **2,** 325, 1901.

[4] BACHMETJEW, *Wiedemann Ann.*, **43,** 723, 1891.

get at what is fundamental. It comes back to saying that our ignorance of the subject of magnetism is the bar to a better interpretation of all these various results.

52. The Peltier Effect between Magnetized and Unmagnetized Portions of the Same Substance.—If an electric current flows through a thermocouple, one junction will become cooler and the other warmer than in the neutral state. This is known as the Peltier[1] effect. It is the reciprocal of the Seebeck[2] effect discussed in Sec. **51.** Such an effect ought to exist for a thermocouple made up of a magnetized and an unmagnetized portion of the same substance. Apparently no one has investigated such an effect. It must be present as its reciprocal is found in the Seebeck effect.

53. The Influence of a Magnetic Field on the Peltier Effect.—Given a thermocouple, Battelli[3] showed that a magnetic field influenced the Peltier effect. In both a longitudinal and transverse field an iron-copper couple increased the Peltier effect as did also a nickel-copper set. Houllevigue[4] has investigated iron and steel relative to the Peltier effect when the elements were magnetized and more recently Borelius and Lindh[5] studied a bismuth-copper couple and found a change. Thus the effect is not limited to ferromagnetic substances alone.

54. The Influence of a Magnetic Field on the Thomson Effect. It was pointed out by Sir William Thomson[6] on theoretical grounds, that there must be thermo-electric effects existing between the different parts of a metal at different temperatures. This he was able to demonstrate experimentally. Thus far, we have seen that any source of an electromotive force seems to be influenced by a magnetic field. It might be supposed that the Thomson electromotive-force would also. Some work has been done on this problem, but very much more needs to be done. The effect of a magnetic field on the reverse Thomson[7] effect should also be considered. All of these thermo-electric effects which are changed by the application of a magnetic field have

[1] PELTIER, *Ann. chim. phys.*, **56,** 371, 1834.

[2] SEEBECK, *Poggendorff Ann.*, **6,** 1, 1826.

[3] BATTELLI, "Atti. R. Ist. Veneto," vol. IV, pp. 1452, 1581, 1637, 1676, 1743, 1893.

[4] HOULLEVIGUE, *Jour. de phys.*, **5,** 53, 1896.

[5] BORELIUS and LINDH, *Ann. der Phys.*, **53,** 97, 1917.

[6] THOMSON, *Philos. Trans.*, **146,** 722, 1856.

[7] FOOTE and HARRISON, *Sci. Jour. Wash. Acad.*, **8,** 545, 1918.

either a direct or indirect bearing on the galvanomagnetic effects discussed earlier in the chapter. Moreau[1] has, for instance, pointed out that a certain definite relation exists between the Nernst, Hall, and Thomson effects. Doubtless, there are many other relations not yet accounted for which only further study of the various effects will bring out. This chapter covers part of a "vast and fascinating domain, wherein dwell some of the mysteries of matter." It is a field that must be investigated further.

[1] Moreau, *Compt. rend.*, **130**, 122, 412, 562, 1900.

CHAPTER V

MAGNETO-THERMICS

55. Thermomagnetic Effects.—Under the caption of magneto-thermics may be discussed: (1) Those phenomena in which a magnetic field influences the trend of ordinary heat phenomena; (2) the development of heat by a magnetic field, and (3) the influence of heat on all magnetic phenomena. In Sec. **47** of the preceding chapter a series of galvanomagnetic effects was discussed. Similarly, a series of thermomagnetic effects will now be reviewed which are schematically illustrated in Figs. 112*e*, *f*, *g*, and *h*. In the galvanomagnetic effects an electric current flowed from the positive to the negative end of the rectangular plate, while in the following effects there is a flow of heat from the hot end of the plate to the cold.

a. Nernst Effect.—In the Nernst[1] effect the flow of heat may be visualized in the same way as the lines of electric flow in the Hall effect. When a magnetic field is applied normally to them, these equipotential lines of heat flow, will be distorted. The two points C and D will no longer be at the same potential and as a result an electric current will flow when the circuit is completed through a galvanometer. One might think of this as a counterpart of the Ettingshausen effect. In the latter, a distortion of the equipotential lines of flow by a magnetic field gave an unequal change in temperature at its lateral edges, while the effect under discussion, a distortion of the equipotential lines of heat flow, gave an unequal change in potential at the lateral edges of the plate. Figures 118 and 119 show the conventional signs for the Nernst effect. Among those who studied the phenomena may be mentioned Everdinger, Barlow, and Buckley.[2]

b. Righi-Leduc Effect.—Figure 112*f* indicates that if a flow of heat takes place along a rectangular plate from left to right,

[1] von Ettingshausen and Nernst, *Wiedemann Ann.*, **29**, 343, 1886.

[2] Everdinger, *Commun.* Phys. Lab., Leiden, **42**, 4, 1898; **48**, 3, 1899; Barlow, *Ann. der Phys.*, **12**, 897, 1903; Buckley, *Phys. Rev.*, **4**, 482, 1914.

the equipotential lines of heat flow will be distorted and an unequal change in temperature occurs at the edge of the plate. This effect was discovered independently by Righi[1] and Leduc.[2]

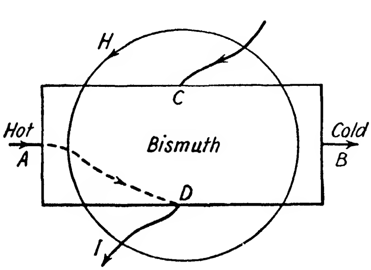

FIG. 118.—In the Nernst effect if the heat tends to flow toward the point of high potential D, the effect is said to be positive. Bismuth is an example.

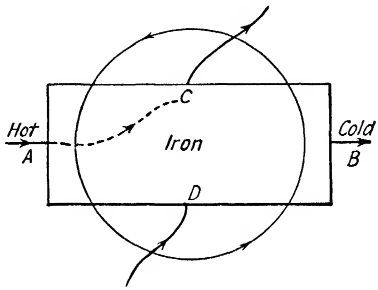

FIG. 119.—In the Nernst effect if the heat tends to flow toward the point of high potential C then the effect is negative. Iron is an example.

The signs for the Righi-Leduc effect are represented by Figs. 120 and 121. A number have investigated this effect and the influence of a change in the magnetizing field and the temperature

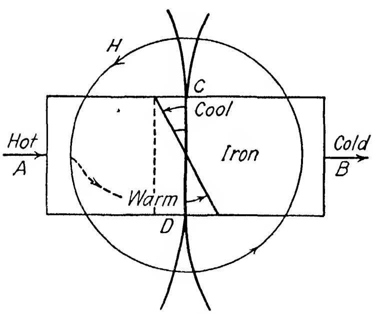

FIG. 120.—The sign of the Righi-Leduc effect is said to be positive when the heat flow from A to D is in the same direction as the current producing the field. Iron is an example.

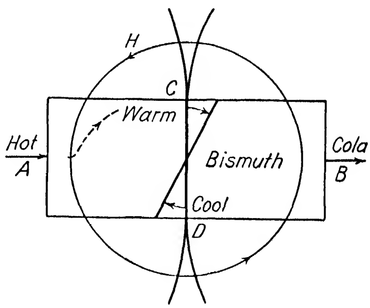

FIG. 121.—The direction of heat flow in a bismuth plate is opposite to that of iron and is, therefore, negative in its sign.

imposed upon the material. Senepa[3] measured the effect in bismuth over a range of magnetizing force, 600 to 1,200 gausses. Smith and Smith[4] did the same for larger field strengths in iron,

[1] RIGHI, *Amer. Jour. Sci.*, **34**, 228, 1887; *Philos. Mag.*, **6**, 725, 1903.
[2] LEDUC, *Compt. rend.*, **104**, 1783, 1887; *Zeitsch. d. phys. Chem*, **2**, 107, 1888.
[3] SENEPA, *Nuov. Cim.*, **6**, 303, 1913.
[4] SMITH and SMITH, *Phys. Rev.*, **5**, 35, 1915.

nickel, and cobalt. Hall and Campbell[1] investigated the effect at two mean temperatures and found a large change. This effect found by Righi and Leduc is the analogue of the Hall effect. Königsberger and Gottstein[2] have worked out a theoretical relation between the two effects.

c. Thermomagnetic, Longitudinal, Potential Difference.—Ettingshausen and Nernst, investigating the various relations between a magnetic field and the flow of electricity and heat, discovered that if heat flows through a rectangular plate as in Fig. 112*g*, and at the same time is magnetized normally to the lines of flow, there will be set up a longitudinal electromotive force between the points *C* and *D*. The effect is said to be positive if the electric current developed is parallel and flowing in the same direction as the heat. Moreau[3] did some very interesting work on this subject, using nickel, soft steel, and iron as the medium of investigation. Smith[4] studied the effect in bismuth through a wide range of temperature.

d. Thermomagnetic, Longitudinal, Temperature Difference.—This is the last of the four thermomagnetic effects. In this effect a magnetic field, applied normally to a rectangular plate through which heat is flowing, changes the difference in temperature between the two points *C* and *D* (Fig. 112*h*). This behavior of the flow of heat in a magnetic field amounts to a change in the thermal conductivity just as in the galvanomagnetic, longitudinal, potential difference the magnetic field changed the electrical conductivity. Metals such as bismuth,[5] tellurium,[6] iron,[7] and nickel[8] show a decrease in their thermal conductivity when exposed to a magnetic field. There is a need here to extend this work to metals like gold and silver, and correlate the work with some of the other effects which have just been discussed.

56. The Influence of Heat on Magnetic Phenomena.—Faraday pointed out the universality of magnetic qualities. It might also be pointed out that there is a universality in the

[1] Hall and Campbell, *Proc. Amer. Acad.*, **46**, 625, 1911.

[2] Königsberger and Gottstein, *Ann. der Phys.*, **46**, 446, 1915; **47**, 566, 1915.

[3] Moreau, *Jour. de phys.*, **10**, 685, 1901.

[4] Smith. *Phys. Rev.*, **2**, 383, 1913.

[5] Lownds, *Philos. Mag.*, **5**, 141, 1903.

[6] Wold, *Phys. Rev.*, **7**, 169, 1916.

[7] Hall and Campbell, *Proc. Amer. Acad.*, **46**, 625, 1911.

[8] Schmaltz, *Ann. der Phys.*, **16**, 398, 1905.

effect of heat on magnetic properties. One may list all of the magnetic phenomena to be thought of and then methodically go through the whole list and show that in one way or another heat influences the phenomena thus catalogued.

For convenience one may classify substances in three main groups, ferromagnetic, paramagnetic, and diamagnetic, and ask: What are the main effects of heat on these various classes of substances?

a. Ferromagnetic Bodies—Critical Temperatures.—When a piece of iron is heated to higher and higher temperatures it loses its power to become magnetic, and at a certain definite temperature,

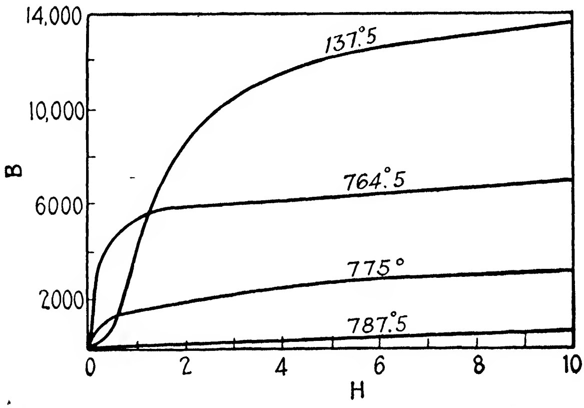

FIG. 122.—The magnetic induction of iron decreases with increasing temperature, until the critical temperature is attained.

called by Hopkinson the "critical temperature," it rather abruptly changes from the ferromagnetic state to the paramagnetic. For a given piece of iron, this critical temperature is a very definite one. So definite is it, that it is an old practice to get a rough determination of temperatures by testing as to whether a piece of iron, which is hot, will still be attracted by a magnet or not. If it is above the critical temperature there will be no attraction. This occurs at about a dull-red heat.

Hopkinson[1] and particularly Morris[2] made some extensive investigations regarding the changes which occur magnetically when the temperature varies. These studies may be best reviewed by giving some of the curves which they have plotted. Figure 122 shows how the values of the magnetic induction vary

[1] HOPKINSON, *Philos. Trans.*, **180**, 443, 1889.

[2] MORRIS, *Philos. Mag.*, **44**, 213, 1897.

as the temperature increases from 137.5° C. to 787.5° C. It is evident that just a little beyond 787.5° C. the magnetic induction is reduced to zero. A still more striking way of showing these transformations is to plot the permeabilities for different field strengths against temperatures. This has been done in Fig. 123. These curves bring out very clearly that at about 787.5° C. a critical temperature has been attained and the iron behaves as

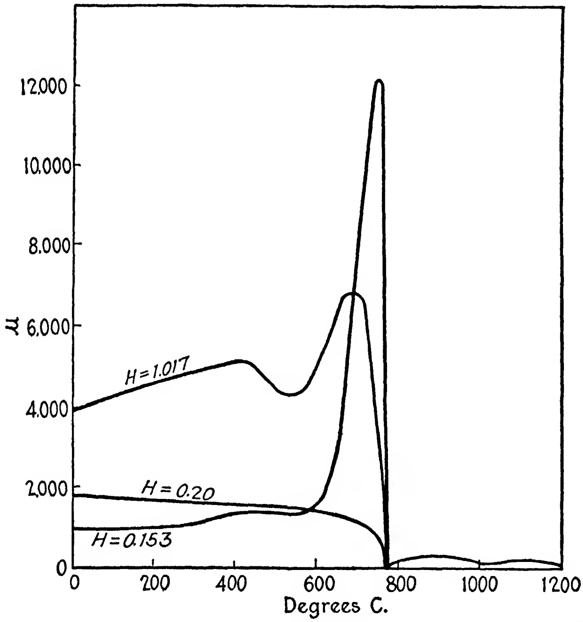

FIG. 123.—The effect of heat on iron is to produce a very abrupt change in the magnetic permeability. H is the magnetizing force.

a non-magnetic body. Berson[1] has shown that the permanent magnetism also disappears at this point. This also fits into the picture given by Fig. 122. It is also known that at this critical temperature other physical properties change radically along with the magnetic characteristics. For instance, at the critical temperature there is an abrupt change in the thermo-electric property of iron.[2] It has been observed that the temperature coefficient of electric resistance in iron is also closely associated with the critical temperature.[3] Barrett[4] found that a steel

[1] BERSON, *Ann. chim. phys.*, **8**, 443, 1886.
[2] TAIT, "Science Papers," vol. I, p. 218.
[3] KOHLRAUSCH, *Wiedemann Ann.*, **33**, 42, 1888.
[4] BARRETT, *Philos. Mag.*, **47**, 56, 1874.

wire heated to a bright-red heat would, as its color reached a dull red on cooling, suddenly brighten up again. This indicated that the cooling process was arrested and actually the temperature increased about 15°. This phenomenon was called recalescence, and the temperature at which it occurred the point of recalescence. This is intimately associated with the critical temperature. That these various effects occur at about the same temperature is evidently due to some process going on within the metal which at the critical temperature affects all of the phenomena mentioned.

In metallurgical parlance, this point where iron loses most of its magnetism is called the A_2 point. It has a definite value for pure iron but varies for different steels. In addition to this marked change in iron, as its temperature is raised, there occur two other points which are of great interest and which may be studied by magnetic methods. These two special points in temperature indicate allotropic transformations in the iron. One is at about 920° C. and the other at 1410° C. An allotropic transformation consists of an internal change from one space lattice to another. Iron has a body-centered cubic lattice below 920° C. and a face-centered cubic lattice between about 920° C. and 1410° C. Below 920° C. iron is called α-iron which is ferromagnetic below 790° C., and paramagnetic above. From 920° C. to 1410° C. it is called γ-iron. Thus iron passes from the ferromagnetic to the paramagnetic state in a continuous fashion. Above 1410° C. until it melts, iron becomes once more (called δ-iron by some) α-iron. However, it is not so magnetic as below 920° C., although it is more magnetic than the γ-iron.

The metallurgist calls the transformation temperature at 790° C. the A_2 point, at 920° C. the A_3 point, and at 1410° C. the A_4 point. If the temperature is rising, the transformation points are called Ac_2, Ac_3, and Ac_4. If the temperature is falling, they are designated as Ar_2, Ar_3, and Ar_4, respectively. When the change in temperature is very slow the transformation points on cooling may be made to very nearly coincide with those on heating. Ishiwara[1] has shown in a particularly interesting way how iron beyond the A_4 point may be thought of as a continuation of the iron below the A_3 point. This is shown in Fig. 124.

[1] Ishiwara, *Sci. Repts. Tôhoku Imp. Univ.*, **6**, 133, 1917; Benedicks, *Jour. Iron Steel Inst.*, No. 1, p. 407, 1914.

The dotted portion of the curve indicates the continuation of the magnetization-temperature curve of α-iron through the γ-region. Fundamentally important is some work by Honda, Masumoto, and Kaya[1] in measuring the magnetization of single crystals of iron at different high temperatures by the ballistic method. Once the point of magnetic saturation is reached for any temperature the magnetic intensity remains surprisingly constant along all of the axes. The allotropic changes of iron can be studied by

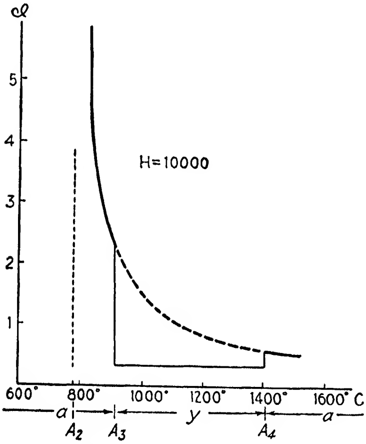

FIG. 124.—Magnetization-temperature curve for iron. (*Ishiwara.*)

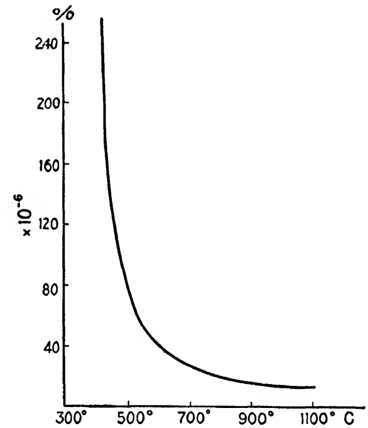

FIG. 125.—Susceptibility-temperature curve for nickel. (*Honda, Takagi.*)

x-ray[2] examination and thus check the magnetic tests. The magnetization-temperature curve for nickel does not indicate allotropic transformations. It does have a point[3] corresponding to A_2 for iron where it does lose its magnetic properties. This occurs at about 390° C. The susceptibility-temperature curve for nickel is shown in Fig. 125. Cobalt has its critical temperature[4] at about 1115° C. This is indicated in Fig. 126. Cobalt also possesses allotropic transformations. The very large field of ferromagnetic alloys does not permit of detailed discussion of the effect of temperature on them. It is evident, however, that

[1] HONDA, MASUMOTO and KAYA, *Sci. Repts. Tôhoku Imp. Univ.*, **17**, 111, 1928.

[2] WESTGREN, *Jour. Iron Steel Inst.*, **103**, 303, 1921;
WESTGREN and PHARGMEN, *Jour. Iron Steel Inst.*, 1922;
DEJEAN, *Ann. de Phys.*, **18**, 171, 1922.

[3] HONDA, *Sci. Repts. Tôhoku Imp. Univ.*, **10**, 433, 1921.

[4] HONDA and SHIMIDZU, *Jour. Coll. Sci.*, **20**, art. 6, 1904.

more and more magnetic methods[1] are being found of great use in following transformations which occur in pure metals and alloys.

Another property, which decreases with increasing temperature until the critical point is reached, is the energy loss in hysteresis. Terry[2] has done some interesting work along this line with electrolytic iron. All of the magnetostrictive effects are profoundly altered by temperature changes.[3] There are many other magnetic phenomena which need further study of the effect of temperature upon them.

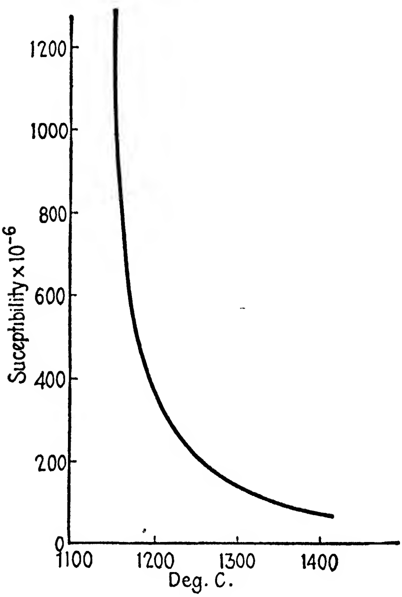

FIG. 126.—Susceptibility-temperature curve for cobalt. (*Honda, Takagi.*)

b. Paramagnetic Bodies.—It has just been seen that bodies pass continuously from the ferromagnetic to the paramagnetic state without allotropic transformations. This has led investigators to assume that paramagnetic bodies would become ferromagnetic, if their temperatures were sufficiently lowered. In working with an alloy of three parts of iron and one part of nickel, Hopkinson[4] discovered that this alloy was non-magnetic at ordinary temperatures, but when cooled to a few degrees below freezing it became as strongly magnetic as the average cast iron. This is a case where the critical temperature lies below room temperature. Weiss and Onnes[5] investigated vanadium, chromium, and manganese at very low temperatures, but found that they were not ferromagnetic. Many writers distinguish between low- and high-temperature effects. Apparently there is no reason for making this distinction other than that much of the

[1] HONDA, *Sci. Repts. Tôhoku Imp. Univ.*, **5**, 285, 1916;
HONDA and MURAKAMI, *Sci. Repts. Tôkohu Imp. Univ.*, **6**, 23, 1917.
[2] TERRY, *Phys. Rev.*, **30**, 136, 1910.
[3] HONDA and SHIMIDZU, *Jour. Coll. Sci.*, **19**, 10, 1903;
SHIMIDZU and TANAKADATE, *Tokyo Sug. But. Kiji*, **3**, 142, 1906.
[4] HOPKINSON, *Proc. Roy. Soc.*, **47**, 23, 1890; **48**, 1, 1890.
[5] WEISS and ONNES, *Commun.* Phys. Lab., Leiden, No. 114, 1910.

research work on magnetic phenomena has been done at room temperature. Any temperature above this is high, and below is low temperature. Fleming and Dewar[1] were pioneers in the field of low-temperature work on magnetic properties. This followed because of their work with liquid air. They found that a soft iron ring immersed in liquid air had its permeability decreased 20 per cent below what it was at 15° C., the magnetizing force being two gausses.

The forces which act on paramagnetic bodies in a magnetic field are so small that ordinary methods for obtaining magnetization curves have to be given up. This holds for diamagnetic bodies as well. Curie[2] pioneered in the development of the technique of this kind of work when he took the old torsion balance of Faraday and made a quantitative instrument out of it. Instead of magnetization-temperature curves, susceptibility-temperature curves were introduced to show the variation of magnetic properties with differences in temperature. In fact, the Curie balance is adaptable to all forms of materials for studying the effect of temperature, as Curie himself studied all forms including iron, nickel, magnetite, oxygen, palladium, bismuth, water, quartz, and many other substances.

Oxygen, palladium, air, glass, porcelain, and $FeSO_4$ were some of the paramagnetic bodies studied by Curie. The glass and porcelain were investigated because they were used as bulbs for holding the specimens. All the others showed a magnetic susceptibility independent of H and inversely as the absolute temperature.

$$\chi = \frac{C}{T}, \tag{170}$$

where $C = 33{,}700 \times 10^{-6}$. This is known as Curie's law for paramagnetic substances. Equation (165) would indicate that at absolute zero χ would become infinite. Dewar and Fleming[3] studied $MnSO_4$ and oxygen down to $-186°$ C. and found that the equation held as far as this temperature. The work of Onnes and Perrier,[4] Oosterhuis,[5] Honda,[6] and Owen[7] seemed to show that

[1] Fleming and Dewar, *Proc. Roy. Soc.*, **60**, 81, 1896.

[2] Curie, *Ann. chim. phys.*, **5**, 289, 1895.

[3] Dewar and Fleming, *Proc. Roy. Soc.*, **60**, 57, 1897; **63**, 311, 1898.

[4] Onnes and Perrier, *Proc. Akad.*, Amsterdam, **16**, 894, 1914; *Commun.* Phys. Lab., Leiden, No. 116, April, 1910.

[5] Oosterhuis, *Proc. Akad.*, Amsterdam, **16**, 432, 1913.

[6] Honda, *Ann. der Phys.*, **32**, 1027, 1910.

[7] Owen, *Ann. der Phys.*, **37**, 657, 1912.

this law of Curie was not at all true for the majority of paramagnetic substances.

c. Diamagnetic Substances.—Figure 127 shows the variation of mass susceptibility of a number of representative diamagnetic materials. Bismuth was particularly interesting, and had special attention paid to it. In spite of the behavior of bismuth, Curie formulated for diamagnetic substances the law, "the magnetic susceptibility of diamagnetic substances is independent of the temperature and the state of the substance." Dushman,[1] in an excellent paper, shows the discrepancies of Curie's two laws by

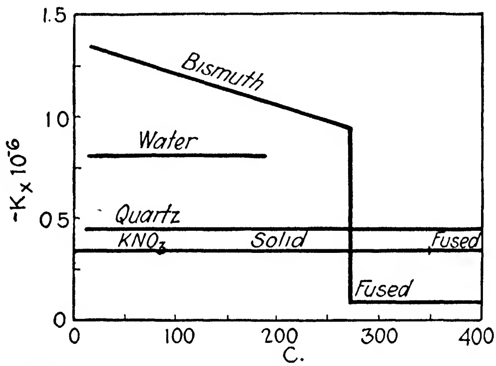

FIG. 127.—Showing the relations which Curie found between mass susceptibilities and temperatures for several diamagnetic substances.

means of two tables. The work since Curie's paper in 1895 has shown so many departures from his laws that it has led Kunz[2] to remark: "It seems to me not justified to maintain Curie's rule, as there are many more exceptions than confirmations. The same is true for diamagnetism . . . There are only very few elements which do not vary within the whole temperature range."

In spite of these criticisms Curie's work remains today a classical research and is the forerunner of many other important researches in this interesting field.

On page 100 was mentioned the enormous amount of work which has been done on the magnetic susceptibility of organic and inorganic compounds. Similarly a large amount of work has been done on the effect of temperature on the magnetic proper-

[1] DUSHMAN, *Gen. Elec. Rev.*, May, August, September, October, and December, 1916.

[2] KUNZ, Eighth Internat. Cong. *Appl. Chem.*, **22**, 187, 1912.

ties of compounds. Chief among the investigators of this field may be mentioned Onnes, Perrier, Oosterhuis, Honda, and Ishiwara.[1]

57. Heat Treatment and Effects on Temporary and Permanent Magnetisms.—Not only is the actual temperature at which bodies are tested of vital importance in the field of magnetism, as we have seen in the preceding sections, but the past thermal history of a substance also plays an important rôle. If a piece of steel has been quenched above the critical temperature, its magnetic characteristics will be different from what they are if the same sample of steel is tempered at different drawing temperatures.

In making magnetos, telegraph and telephone instruments, and particularly the fine ammeters and voltmeters for precision work, it is highly important to have *permanent magnets which remain permanent.* The qualities desired in such magnets will first rest on the kind of steel or alloy employed, but finally they will all depend upon the heat treatment accorded the steel or alloy selected. Seven per cent tungsten and chromium steels are the most widely used steels for permanent magnets.[2] In 1920 Honda[3] discovered a cobalt steel which made excellent permanent magnets and which had remarkably high coercive force. If, however, these various steels are not properly heat treated they will be worthless for use in making permanent magnets.

Spooner[4] summarizes the effect of heat treatment by saying;

In order to be highly magnetic, steel must be in the α state. Also, in order to have a high coercive force, the carbon or carbides must be in solution. To change from the γ to the α state tends to throw these carbides out of solution and thereby gives a poorer magnet. It is the function of heat treatment [quenching and, perhaps, drawing] first to put the carbides in a solution by applying a sufficiently high temperature for a sufficiently long time [in the γ region], and then cool the material at a sufficiently rapid rate to keep the carbides in solution, but still in such a way as to retain as little as possible of the material in the γ state.

[1] Onnes, Perrier, and Oosterhuis, *Commun.* Phys. Lab., Leiden, Nos. 116, 122a, 124a, 129b, 132e;

Honda and Ishiwara, *Sci. Repts. Tôhoku Imp. Univ.*, **3**, 303, 1914; **3**, 139, 1914; **4**, 215, 1915.

[2] Moir, *Philos. Mag.*, **28**, 738, 1914.

[3] Honda and Saito, *Phys. Rev.*, **16**, 494, 1920.

[4] Spooner, "Properties and Testing of Magnetic Materials," p. 72, 1927.

As pointed out by Evershed:[1]

> The carbides tend to go out of solution and enter the crystalline state. Due to the very low mobility of the molecules at room temperature, however, this is an extremely slow process. It does, nevertheless, go on for years, resulting in the well-known ageing or weakening of the magnets. It is obvious why a higher temperature hastens the ageing process. It is due to the fact that the molecular mobility increases with increasing temperature.

Tungsten and carbon-steel magnets are quite effectively aged by steaming or holding at a slightly lower temperature for 24 hours. The permanency attained in the ageing process is best illustrated by the accuracy maintained in a good ammeter or voltmeter. The recent discovery of permalloy,[2] an alloy of nickel and iron, has emphasized the very great importance of heat treatment for bringing out the desirable qualities of this metal, *viz.*, a high permeability for low magnetic fields. With proper treatment, the "initial" permeability may have values around 10,000 at field strengths of 0.001 to 0.002 gauss. At field strengths like the earth's (0.18 gauss), saturation is practically attained.

One of the more unusual alloys is the ternary one called the Heusler[3] alloy. The one showing the greatest intensity of magnetization has a composition of Al 15 per cent, Mn 23.5 per cent, Cu 61.5 per cent. Its magnetic permeability, after proper heat treatment, is about that of poor cast iron. "The best heat treatment seems to be to quench the material and then anneal it for a long time at a temperature slightly over 100° C." This alloy has attracted a great deal of attention because its constituents are non-magnetic and the hope was entertained that it might be very helpful in developing a comprehensive theory of magnetism. While we are on the subject of alloys and their heat treatment, we should mention one other alloy, *viz.*, monel metal.[4] This is an alloy of approximately 67 per cent of nickel, 28 per cent of copper, and 5 per cent of other metals which is

[1] Evershed, *Jour. Inst. Elec. Eng.*, **63**, 725, 1925.

[2] Elmen and Arnold, *Jour. Franklin Inst.*, **195**, 621, 1923.

[3] Heusler, Stark and Haupt, *Verh. d. Phys. Gesellsch.*, **5**, 219, 1903; Knowlton, *Phys. Rev.*, **32**, 54, 1911.

[4] Burrows, *Elec. World*, **76**, 115, 1921;
Williams, *Phys. Rev.*, **29**, 370, 1927;
Inglis, *Instruments*, **2**, 129, 1929.

found in nature in about these proportions. Its critical temperature is about 100° C. as is shown by Fig. 128. The magnetostrictive effect is particularly interesting because, if a rod of monel metal is placed in a solenoid and the field is increased from zero to about 1,200 to 1,500 gausses and maintained at that value, it will keep on shortening for a long time (4 to 5 minutes), after variation in the magnetic field has ceased. This after-effect seems to be some function of previous heat treatment.

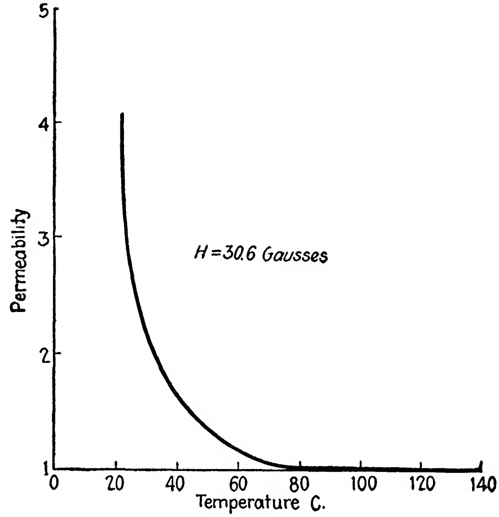

FIG. 128.—The permeability of monel metal as a function of temperature. (*Inglis.*)

58. Change in Temperature Due to Magnetization.—In Sec. **28** the energy loss due to hysteresis was discussed. This dissipation of energy indicated an increase of temperature which may be calculated according to Joule's law,

$$\text{Work} = J \times \text{heat} = \text{ergs}, \tag{171}$$

where the heat is expressed in calories and J is the mechanical equivalent of heat. The specific heat of iron is 0.116 and the density is 7.6. The heat which accumulates in a cubic centimeter of iron during the process of magnetization will be:

$$\text{Heat} = 7.6 \times 0.116 \times \text{change in temperature}$$

$$W_v = J \times \text{Heat}_v, = (4.2 \times 10^7)(7.6 \times 0.116 \times t) \text{ ergs} \tag{172}$$

$$= \frac{1}{4\pi}\int H dB \text{ ergs/ccm/cycle.}$$

For a single cycle, therefore, the rise in temperature will be:

$$t = \frac{HdB}{4\pi(4.2 \times 10^7)(7.6 \times 0.116)}C. \tag{173}$$

For dynamo magnet steel $HdB/4\pi$ is equal to 500 ergs/ccm/cycle. Therefore,

$$t = \frac{500}{(4.2 \times 10^7)(7.6 \times 0.116)} \tag{174}$$
$$= 1.35 \times 10^{-5} \text{ degrees per cycle.}$$

This is a very small change in temperature and would not influence the results obtained in measuring the changes in length due to a magnetic field.

59. Pyro- and Piezo-magnetism.—Certain crystals, which at ordinary temperatures exhibit no electric charges, will on uniform heating develop unlike surface electrification on opposite surfaces of the crystal. Some crystals exhibit the same phenomenon when they are stretched. The first is called pyro-electricity and the second piezo-electricity. Because early experimenters were not able to find the two corresponding magnetic phenomena, it became a more or less general idea that no such effect could exist. With the advent of the electron theory of matter, however, physicists began to inquire as to the possibility of the phenomena of pyro- and piezo-magnetism.

Voigt[1] was one of the earliest to investigate both theoretically and experimentally the presence of these effects. He found that theoretically they should be present in certain mono- and triclinic crystals as well as in certain rhombohedral, hexagonal, and tetragonal systems. Experimentally he had no success and, although he was able to detect a difference in pole strength of the order 10^{-7}(g.cm.sec.), he was not able to find what he felt sure was an effect of either pyro- or piezo-magnetism.

Voigt's general method was to heat or stretch a cylinder of the crystal under examination near a very sensitive astatic magnetometer, but outside disturbances or impurities of the crystal seemed to blanket any real effects. All Voigt could do was to set upper limits to the size of the effect, greater than which he was sure it could not be.

Inasmuch as a permanent magnetic moment cannot be compensated as is possible with an electric moment by means of induced surface charges, a pyromagnetic crystal must show a

[1] Voigt, *Drude's Ann.*, **9**, 94, 1902.

magnetic moment at room temperature. Even this Voigt was not able to identify positively. This is a problem that ought to be tried with more sensitive instruments than Voigt had.

60. Change in Boiling Point and Specific Heats Due to Magnetization.—duBois[1] has considered the possibility of a change in the boiling point of a substance when subjected to a magnetic field. The equation for the change in boiling point indicated that paramagnetic substances should show an increase and, contrariwise, diamagnetic bodies should show a lowering. The calculated increase in boiling point for oxygen amounted to only $\frac{1}{100}$ of a degree. It is a very small effect and yet Nagaoka[2] in an amalgam with which he worked found a change of $\frac{1}{20}$ of a degree.

From the law of conservation of energy Stefan[3] calculated that the specific heat of magnetized iron should be greater than for the umagnetized. This turns out to be an exceedingly small change and, at first, it seems not to have been observed in iron under ordinary circumstances. However, when iron was alloyed with nickel, Hill[4] found that the specific heat of this alloy was less when magnetized than in the unmagnetic state and that the difference decreased with increasing temperature. Later Weiss and Beck[5] measured the specific heat of iron, nickel, and magnetite at various temperatures and found a change during the magnetic transformation.

[1] duBois, *Verh. d. phys. Gesellsch.*, **17,** 148, 1898.

[2] Nagaoka, "Winkelmann's Handbuch der Physik," vol. 5, p. 372.

[3] Stefan, *Wiener Berichte*, **64,** 219, 1871.

[4] Hill, *Verh. d. Deutsch. phys. Gesellsch.*, **3,** 113, 1901; *Science*, Oct. 18, 1901.

[5] Weiss and Beck, *Jour. Phys.*, **7,** 249, 1908.

CHAPTER VI

MAGNETO-OPTICS

The term magneto-optics has been in use for many years. It usually refers only to those phenomena in which a magnetic field influences the behavior of light. In this chapter the term magneto-optics will be used to include the effect of light on magnetic properties, if such effects exist.

61. Faraday Effect.—Faraday seemed to have had an intuitive sense that certain relations must exist between certain physical phenomena. One of these was that magnetism must have an influence on light. This he sought for diligently and could not find it, because as we know now, his equipment was not sensitive enough. He did, however, uncover another very important phenomenon, *viz.*, that a transparent, isotropic medium, when placed in a strong magnetic field, has the power to rotate the plane of polarization of light, when the light moves parallel to the magnetic lines of force. This effect has been observed in solids,[1] liquids,[2] and gases.[3] Among the first substances examined it happened that the direction of rotation was always the same, *i.e.*, in the same direction as the electric current would flow in a solenoid to give the direction of the magnetic field being used. Later it was discovered that some substances rotated the plane of polarization in the opposite direction. Consequently the direction of rotation first observed was called positive and the latter negative. For a while it appeared that diamagnetic bodies showed only a negative rotation and paramagnetic the opposite. However, even this proved to be an inexact division and now all substances are classified under four heads: positive and negative diamagnetic bodies, and positive and negative paramagnetic substances.

There are substances which rotate the plane of polarization naturally (sugar, for instance) without applying a magnetic field. The manner of rotation in the natural rotation is quite

[1] Faraday, "Experimental Researches," vol. III, p. 1, 1845.

[2] Rodger and Watson, *Philos. Trans.*, *A*, **186**, 621, 1895.

[3] Kundt and Roentgen, *Wiedemann Ann.*, **6**, 332, 1879.

different from that of the magnetic. If, in the case of natural rotation, the light is reflected back through the body so as to retrace its path, the two beams will be rotated in opposite directions and just balance each other. In magnetic rotation the going and returning rays add their effects giving a total rotation double that of the single passage.[1]

In Fig. 129 is shown a simple arrangement for observing and measuring the Faraday effect. A solenoid is used as a source of magnetomotive force. Two Nicol prisms are set in crossed position, one at each end of the solenoid. When the prisms are crossed no light passes. When the block of glass within the solenoid is magnetized, the plane of polarization is rotated in the glass, and light goes through the system. The

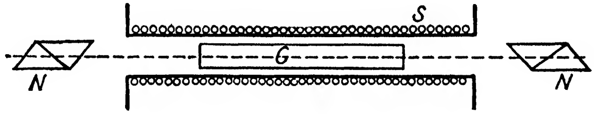

FIG. 129.—An arrangement for studying the Faraday effect in a transparent substance. G might be a block of glass. S is the solenoid for producing a magnetic field and N, N are the crossed Nicol's prisms.

light may be shut off again by turning the analyzing Nicol through a certain definite angle. The amount of rotation of the Nicol to produce extinction is the angle through which the plane of polarization has been rotated. Instead of a solenoid an electromagnet may be used as shown in Fig. 29.

Verdet[2] did a great deal of experimenting in this field; so much so that the amount of rotation characteristic of a substance is now called Verdet's constant. The rotation may be expressed by the equation

$$R = C(V_a - V_b), \tag{175}$$

where R is the rotation in minutes, V_a and V_b the magnetic potentials at the ends of the path considered, and C is *Verdet's constant* which is characteristic of the substance. Verdet's work was done before the value of absolute units was realized, so he expressed his constant in terms of that for water. Gordon[3] was the first one to work out the absolute value of Verdet's constant for sodium light passing through carbon bisulphide. Lord Rayleigh,[4]

[1] FARADAY, "Experimental Researches," vol. III, p. 453.

[2] VERDET, "Collected Papers," vol. I, p. 112, 1872.

[3] GORDON, "Electricity and Magnetism," vol. II, p. 218.

[4] LORD RAYLEIGH, "Science Papers," vol. II, p. 360.

using more sensitive and accurate methods, particularly in the analyzer, worked out the same absolute value.[1]

The amount of rotation of the plane of polarization in a magnetic field depends upon the wave length. If white light is used, different colors will appear in the field of view as the analyzer is rotated. Such a display of colors is spoken of as *rotary dispersion.* Theory shows that if a substance possesses anomalous dispersion it will also exhibit anomalous magnetic rotary dispersion. Substances like a solution of didymium salt show this. Bates[2] and Wood[3] have done some interesting work along this line.

A linear vibration may be regarded as the resultant of two oppositely directed circular vibrations. In the case of optically active substances, Fresnel[4] explained the phenomenon by saying that when plane-polarized light entered such a medium it was broken into two oppositely circularly polarized rays which were propagated with different velocities. If this difference in speed puts one component half a wave length ahead of the other in going through a certain thickness of the medium, the plane of polarization of the emergent beam will have rotated 90° with respect to the entrant beam. A very similar explanation holds for the Faraday effect. Righi[5] and also Becquerel[6] showed that right- and left-handed, circularly polarized light traveled with different velocities in a magnetized medium. Brace,[7] in a very brilliant piece of work, showed that plane-polarized light, when it entered a magnetized medium, was also resolved into two circular components. Since Righi and Becquerel had shown that these must travel with different velocities, it was obvious that the emergent ray should have its plane turned with respect to the entrant ray just as in ordinary rotary polarization. The velocities of right- and left-handed, circularly polarized beams of light in a magnetized medium were measured by Mills,[8] who found that the accelerated ray was the one in which the direction of the circular vibration was the same as that of the Amperian currents.

[1] See Table IV, Appendix.

[2] Bates, *Ann. der Phys.*, **12,** 1901, 1903.

[3] Wood, *Philos. Mag.*, **8,** 993, 1904.

[4] Fresnel, "Collected Papers," vol. I, p. 743.

[5] Righi, *Nuov. Cim.*, p. 3, 1878.

[6] Becquerel, *Compt. rend.*, **88,** 334, 1879; **125,** 679, 1897.

[7] Brace, *Philos. Mag.*, **1,** 464, 1901.

[8] Mills, *Phys. Rev.*, **18,** 65, 1904.

One other highly interesting phase of the Faraday effect is the rotation of the plane of polarization in thin films of ferromagnetic substances. Kundt[1] deposited *thin films* of iron, cobalt, and nickel on glass. Since these were less than a wave length of sodium light in thickness, transmission was possible. The rotation was positive and of such a magnitude that if transmission had been possible through one centimeter of iron the rotation for one of the red rays would have amounted to 200,000°. The rotation was less for short waves than for red.

It has been a problem for many years as to whether there was a time lag between the magnetomotive force applied and the rotation of the plane of polarization. Villari[2] performed some experiments which he thought indicated that there was a lag. Blondlot[3] and Lodge[4] took exception to his results and showed that the effects were due to other causes. Recently, however, some very definite results[5] have been obtained by which actual time limits may be set to this lag.

The Faraday effect in a block of glass may be used as a means for measuring the magnetic-field strength of an unknown field if the Verdet constant of the glass is known. A simple arrangement is shown in Fig. 133, in combination with an outfit for observing the Kerr effect. There has been a very large amount of work done on the Faraday effect. There is still much more to be done in correlating it with ordinary rotary polarization, with pressure effects, and with mechanical rotary polarization.[6]

62. Kerr Effect.—When plane-polarized light is reflected from the surface of a polished, non-magnetic metal the light will, in general, be found to be elliptically polarized and the reflected light cannot be extinguished by the analyzer. If, however, the incident light is plane polarized either in or at right angles to the plane of incidence, then the reflected light is plane polarized and can be extinguished by the analyzer. Kerr[7] examined the light, reflected from a polished pole of a strong electro-magnet, under the conditions that one should expect to receive plane-polarized

[1] KUNDT, *Philos. Mag.*, **18**, 308, 1884.
[2] VILLARI, *Poggendorff Ann.*, **149**, 324, 1873.
[3] BLONDLOT, *Compt. rend.*, **94**, 1590, 1882.
[4] LODGE, *Philos. Mag.*, **27**, 339, 1889.
[5] BEAMS and ALLISON, *Phys. Rev.*, **29**, 161, 1927; ALLISON, *Phys. Rev.*, **30**, 66, 1927.
[6] EWELL, *Amer. Jour. Sci.*, **8**, 89, 1899.
[7] KERR, *Philos. Mag.*, **3**, 321, 1877; **5**, 161, 1878.

light in the reflected beam, and found that when the electromagnet was excited the reflected light was no longer plane polarized. It could not be extinguished. Figure 130 shows one arrangement used by Kerr in which the reflecting surface is normal to the lines of force. S' is the source of light, P the polarizer, A the analyzer, and N and S the poles of the electromagnet. In order to get as intense a field as possible at the point of reflection the pole S of soft iron is formed into a blunt wedge with a rounded edge. With this form of pole piece Kerr worked with incident angles from 0° to 80°. His method was to set for almost complete extinction with the magnetizing current off, then, when the magnetic field was applied, the field of view in the analyzer

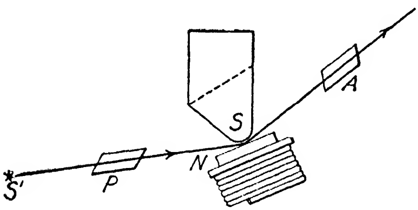

FIG. 130.—An arrangement for studying the Kerr effect at different angles of incidence.

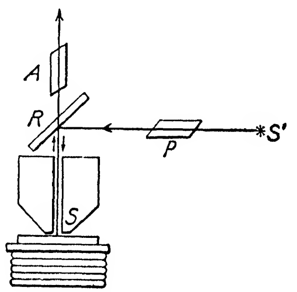

FIG. 131.—Arrangement of apparatus for studying the Kerr effect when the light is incident normally upon the reflecting surface.

would either increase in brightness or grow darker. The rotary effect from magnetized iron was negative, *i.e.*, in a direction opposite to that of the magnetizing current. Kundt[1] extended Kerr's experiments to incident angles which ranged from 0° to 90°. Kundt also found that not only did the angle of incidence vary the effect, but also the angle which the plane of polarization made with the angle of incidence. In order to obtain normal incidence, Kerr used pole pieces as shown in Fig. 131. The pole S was a truncated cone through which a small hole was bored axially. S' is the source, P the polarizer, A the analyzer, and R a reflecting glass plate to throw the light vertically down on the pole N, and allow it to be reflected back up through R and A.

Kerr tried out one other condition, *viz.*, that shown in Fig. 132 in which the lines of force are parallel to the surface. The

[1] KUNDT, *Sitz.-ber.*, Berlin, July 10, 1884; *Philos. Mag.*, **18**, 308, 1884.

rectangular block B is an armature with a polished surface laid across the poles of the electromagnet, whose coils are indicated by the dotted circles. The results he obtained were: When the light was polarized in the plane of incidence, the magnetic rotation was negative for all angles of incidence; when the light was polarized normally to the plane of incidence the rotation was

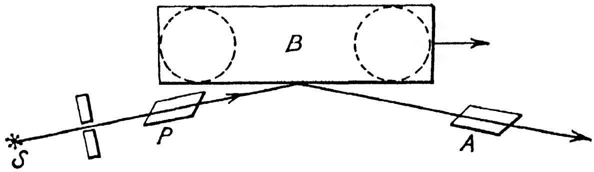

FIG. 132.—In the preceding figures of the Kerr experiment, the magnetic lines of force have been normal to the reflecting surface. In this figure the magnetic lines of force are parallel.

negative for all incident angles between 90° and 75°, disappearing at 75° and then reappearing as a positive rotation for all angles between 75° and 0°. Kerr found no magneto-optical effect for the special case where the wave front is parallel to the lines of force.

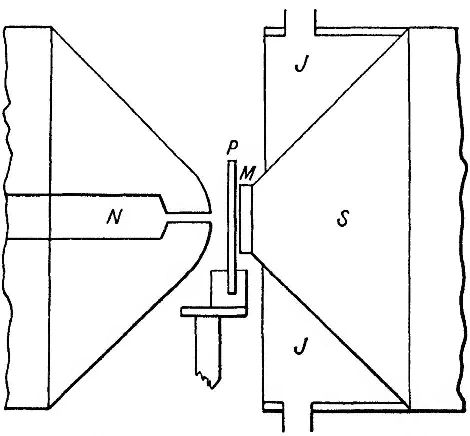

FIG. 133.—duBois' arrangement for studying both the Faraday and the Kerr effect.

These various effects found by Kerr may be explained on the basis that when light strikes a reflecting surface, it penetrates a very short distance and the rotation which occurs is really a Faraday effect in a thin film. Kundt has examined this point of view and it seems to fit with all the facts in the case.

duBois[1] made a very careful study of the Kerr effect when the light is reflected from small, optically flat surfaces, ground on ellipsoids of iron, steel, nickel, and cobalt. The ellipsoids were magnetized by means of a solenoid. duBois found a very simple relation, *viz.*, that the rotation of the polarized ray is proportional to the intensity of magnetization,

$$R = K\mathcal{J}, \tag{176}$$

where K is known as the Kerr constant which is constant for any particular metal and wave length. Once this relation is known this effect becomes a method for measuring strong magnetic fields. duBois developed it in the form shown in Fig. 133. N and S are the conical pole pieces, one of which has a hole through it as shown in Fig. 131. M is the metal plate whose constant is known, and by means of which the electromagnet is to be calibrated. J is a steam jacket surrounding the pole for varying the temperature of the test plate. P is a glass plate with a silvered back. The Verdet constant was known for this plate so that both the Faraday and the Kerr effect could be measured and compared in determining the field strength of the magnet.

63. Magnetic Double Refraction.—Righi and Becquerel showed that right- and left-handed, circularly polarized light traveled with different velocities in a magnetized medium when the direction of propagation was parallel to the magnetic field. This means that, if the velocities are different, there will be a different refracting angle for the two, or a form of double refraction produced magnetically. This, however, is the double refraction which accounts for the magnetic rotation found in the Faraday effect. On the other hand, if the light travels normally to the magnetic field, a slight difference in the periods of the two circularly polarized vibrations still exists. Voigt[2] has shown that this gives a small difference in velocity between the two vibrations, which is a quantity of a higher order than that which accounted for the rotation in the Faraday effect. This effect, normal to the magnetic field, is usually called magnetic double refraction. Voigt and Weichert[3] verified this theory by finding the effect

[1] DUBOIS, *Philos. Mag.*, **29**, 293, 1890; *Wiedemann Ann.*, **39**, 25, 1890; **46**, 545, 1892.

[2] VOIGT, "Magneto-Optik" in Graetz, "Handbuch der Elektrizität und Magnetismus," vol. IV.

[3] VOIGT and WEICHERT, *Wiedemann Ann.*, **67**, 345, 1899.

in a medium of sodium vapor. Zeemann and Geest[1] have studied the same phenomena with good success. Magnetic double refraction has been observed in colloidal solutions[2] and in pure liquids.[3] Havelock[4] pictures a medium with molecules differently spaced along and perpendicular to the field. On the basis of such a point of view he builds up a theory of magnetic double refraction. Cotton and Mouton have a theory that the molecules have definite axes which are oriented along the lines of force.

64. Zeemann Effect.—That intimate relation between magnetism and light for which Faraday sought so diligently was finally observed by Zeemann.[5] The theoretical studies of Lorentz[6] were a great help to Zeemann in finding the effect of a magnetic field upon a vibrating electron. The experiment performed by Zeemann was to examine the light from a sodium flame when it was located between the poles of a powerful electromagnet. Two conditions are of interest; first, when the light is emitted parallel to the magnetic field and, second, when the light passes out normal to the field. Using an electromagnet with a hole through the pole pieces as shown in Fig. 29, the sodium light passed through the hole and into a high-powered spectroscope. The sodium lines were sharp before the field was applied, but, magnetizing the sodium flame, the lines seemed to broaden. This widening was about $\frac{1}{40}$ of the distance between the D lines when the field was about 10,000 gausses. Improved methods indicate that this widening was really the formation of the original line into a doublet, when the radiating source is viewed parallel to the magnetic field. The same line is transformed into a triplet when the radiation is observed normal to the field (Fig. 134). These effects, then, are a shifting of the spectral lines due to a magnetic field, or the Zeemann effect establishes the fact that the vibration period of a monochromatic

[1] Zeemann and Geest, *Proc. Akad.*, Amsterdam, Jan. 25, 1905.

[2] Wood, "Physical Optics," 1st ed., p. 430, 1905;
Kerr, *Brit. Assn. Repts.*, p. 568, 1901.

[3] Cotton and Mouton, *Jour. de phys.*, **1**, 5, 1911;
McComb and Skinner, *Phys. Rev.*, **29**, 525, 541, 1909.

[4] Havelock, *Proc. Roy. Soc.*, **77**, 170, 1905–1906; **80**, 28, 1908.

[5] Zeemann, *Philos. Mag.*, **43**, 228, 1897;
"Researches in Magneto-optics," 1913.

[6] Lorentz, "Theorie der Magneto-optischen Phänomene," in "Encyclopädie der Mathematischen Wissenschaften," no. 5, Heft 2, Teubner, Leipzig, 1909.

radiation is altered by a magnetic field. The circularly polarized doublet emitted from the monochromatic source along the lines of force and the plane-polarized triplet, emitted at right angles to the field, may be taken as the normal types of the Zeemann effect.

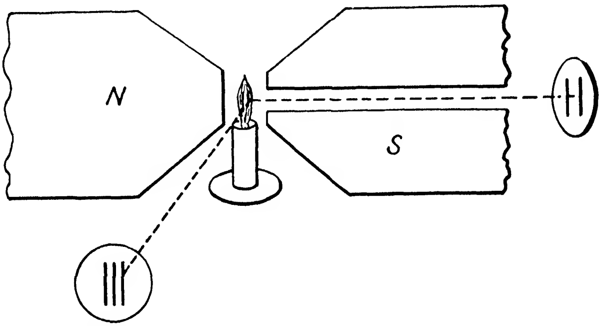

FIG. 134.—The Zeemann effect. A magnetic field impressed upon a luminous vapor changes the wave length emitted.

Further study has shown other types which only our modern atomic theories are attempting to explain.

A physical interpretation of the Zeemann effect is helped by an orbital theory of atomic structure. In Sec. **40** we saw that a

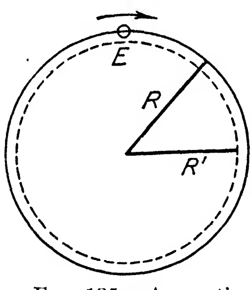

FIG. 135.—A negative charge revolves clockwise in an orbit normal to a magnetic field. When this field is directed from the reader the charge will tend to swing in a smaller orbit.

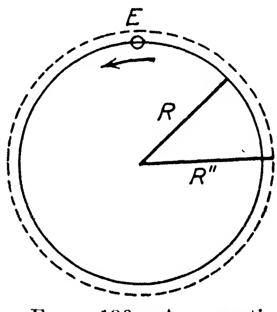

FIG. 136.—A negative charge revolves counterclockwise in an orbit normal to a magnetic field. When this field is directed from the reader the charge will tend to swing in a larger orbit.

conductor, carrying an electric current at right angles to a magnetic field, had a mechanical force exerted upon it. Suppose a negative electron is revolving clockwise in an orbit of radius R, as shown in Fig. 135. When a magnetic field is applied,

normal to the plane of the orbit and away from the reader, there will be a mechanical force exerted upon the electron. This force will cause the electron to revolve in an orbit of reduced radius R', indicated by the dotted line. If the electron revolves counter-clockwise, as in Fig. 136, the electron will seek an orbit of greater radius than R, *viz.*, R''.

In making this change the moment of momentum is conserved. For the radius R' the orbital velocity will be increased, and for the radius R'' it will be decreased. This means that the period of the electron will be less in the smaller orbit than in the larger one.

The mechanical force on the revolving electron E is Hev where H is the magnetic field strength applied, either away from or toward the reader. e is the charge revolving with an orbital velocity of v.

In the absence of the magnetic field H the central force holding the charge in its orbit is

$$f = kR, \tag{177}$$

where R is the radius of the undisturbed orbit. This force is balanced by the equal and opposite force,

$$f = \frac{mv^2}{R}. \tag{178}$$

Let T be the period of the charge in the orbit, and

$$v = \frac{2\pi R}{T}, \tag{179}$$

hence,

$$kR = \frac{m}{R}\frac{4\pi^2 R^2}{T^2}, \tag{180}$$

and

$$k = \frac{4\pi^2 m}{T^2}. \tag{181}$$

If T_1, R_1, and v_1 are the values of T, R, and v when the electron revolves clockwise, then T_2, R_2, and v_2 will stand for the same quantities when the electron revolves counter-clockwise. For the first case,

$$\frac{mv_1^2}{R_1} = kR_1 + Hev_1, \tag{182}$$

and for the second case,

$$\frac{mv_2^2}{R_2} = kR_2 - Hev_2.$$

The respective velocities, v_1 and v_2, may be replaced by their corresponding values given by (179). Also k may be replaced by the value found in (181). When both equations in (182) have been divided by R_1 and R_2, respectively, there follows:

$$\frac{4\pi^2 m}{T_1^2} = \frac{4\pi^2 m}{T^2} + He\frac{2\pi}{T_1}$$

$$\frac{4\pi^2 m}{T_2^2} = \frac{4\pi^2 m}{T^2} - He\frac{2\pi}{T_2}. \tag{183}$$

Subtracting the last equation from the preceding,

$$4\pi^2 m\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right) = 2\pi He\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$$

$$2\pi m\left(\frac{1}{T_1} - \frac{1}{T_2}\right) = He$$

$$\frac{e}{m} = \frac{2\pi}{H}\frac{(T_2 - T_1)}{T_1 T_2}.$$

This was one of the early methods for determining the ratio of the elementary charge to its mass. Since T_1 does not differ very much from T_2 we may write

$$\frac{e}{m} = \frac{2\pi}{H}\left(\frac{T_2 - T_1}{T_0^2}\right). \tag{184}$$

In all wave motion,

$$\lambda = VT. \tag{185}$$

If λ_0 is the wave length emitted by the monochromatic source of light when there is no magnetic field, and λ_c and λ_a the wave lengths emitted when the rotation is clockwise and anti-clockwise in a magnetic field respectively, then

$$\frac{e}{m} = \frac{\frac{2\pi}{H}\left(\frac{\lambda_c}{V} - \frac{\lambda_a}{V}\right)}{\left(\frac{\lambda_0}{V}\right)^2} \tag{186}$$

$$= \frac{2\pi}{H}\left(\frac{\lambda_c - \lambda_a}{\lambda_0^2}\right)V. \tag{187}$$

If the negative charge is revolving, the anti-clockwise rotation will have the greater wave length. If the positive charge is revolving, the clockwise rotation will have the longer wave length. When looking at the source of light parallel to the

field, Zeemann observed that the two lines were circularly polarized. The period of one was $T - dT$, and the other $T + dT$. By experimental methods it was found which one was left-handed, circularly polarized light, and which one was right-handed. Knowing the wave length for each, it could be determined whether it was a positive or a negative charge which was revolving. In every case it has been found that the negative charge is the movable one.

In the case of the triplet, when observed normal to the field, Zeemann found that the three were plane polarized.

It would lead too far afield to discuss the abnormal Zeemann effects. An immense amount of work has been done in this field. There is still great need for investigations in which high magnetic-field intensities are used and observations made with spectrometers of high-resolving power.

Modern spectroscopy has given us more insight into the mechanics of the atom than any other field of knowledge. Surely the subject which combines magnetism and light in a study of atomic structure will be one of the very important fields of investigation in the future.

CHAPTER VII

COSMICAL MAGNETISM

65. Terrestrial Magnetism.—It seems to have been known to Europeans, about the twelfth century of our present era, that the lodestone could be used for giving direction. In a general sort of a way they knew that there was a magnetic field about the earth which gave direction to the lodestone. As to the source or cause of that magnetic field there was, as there is today, very little exact knowledge concerning it. The first real guess made at this question was by Dr. William Gilbert[1] who published in 1600 his famous work "De Magnete." In this treatise concerning the properties of magnetic bodies he deals with "The Earth A Great Magnet." Today we are still raising the question, "Is the earth a magnet or an electromagnet?"[2] The earth possesses a magnetic field having magnitude and direction. Is that field due to the earth being a permanent magnet, or are there earth currents or atmospheric currents which produce the magnetic field of the earth? Some very interesting replies to these questions will be taken up and discussed later. No matter what the cause or causes of the earth's magnetic field may be, the importance of knowing its direction and magnitude has grown ever since man learned that by means of this same field he could direct himself over the surface of the globe.

66. Elements of the Earth's Magnetic Field.—If a long, slim magnet is mounted so that it is perfectly free to turn in any direction about a vertical axis, it will set itself parallel to the earth's field, which gives us the direction approximately north and south (*OH*, Fig. 137). The vertical plane in which the magnet stands is called the plane of the magnetic meridian for the particular point where the magnet is suspended (*VOH*, Fig. 137). This plane, in most places on the earth, forms an acute angle with the plane of the geographical meridian. This angle is called the *angle of declination* or just *declination*. If a compass

[1] GILBERT, Translation of "De Magnete" by the Gilbert Club, p. 211, 1900.

[2] BAUER, *Terr. Mag. Atmos. Elec.*, **16**, 39, 1911.

needle is allowed to turn freely about a vertical axis only, then the angle, which the compass needle makes with the geographical meridian at that place, is this same angle of declination. The magnet, which is free to turn in any direction, will, in general, form an angle with the horizontal. This is known as the *angle of inclination* or just *inclination* or *dip*. If the declination and the dip of any place are known, the direction of the earth's magnetic field is completely determined. Direction and magnitude determine any vector quantity. The intensity of the earth's magnetic field will be determined just as any other field, *viz.*, by the force it exerts upon a unit magnetic pole at the point under question. The intensity of the earth's field is spoken of as the *total intensity*. Since the important part of the earth's field is the horizontal component (in directing instruments of navigation, etc.), it is common practice to resolve the total intensity into the horizontal and vertical components. These are designated as H and V respectively in Fig. 137. If Θ is the angle of dip, then the relation exists between the two components of the total intensity which is given by the equation,

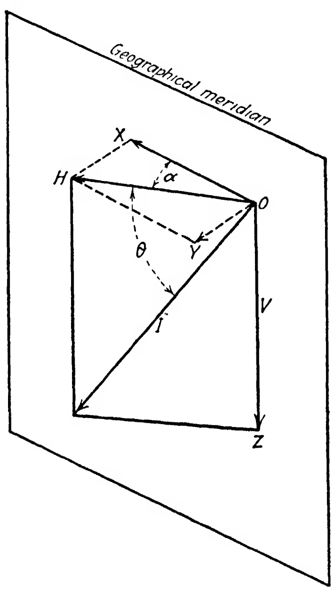

FIG. 137.—The relation between the geographical meridian and the components of the earth's magnetic field.

$$\frac{\text{Vertical component}}{\text{Horizontal component}} = \text{tangent of angle of dip.}$$

$$\frac{V}{H} = \tan \Theta. \qquad (188)$$

The elements of the earth's magnetic field with which the magnetician is most concerned are:

a. Declination.
b. Inclination.
c. Horizontal intensity.
d. Vertical intensity.

67. Experimental Determination of the Earth's Magnetic Elements. *a. Declination.*—Declination was the first of the

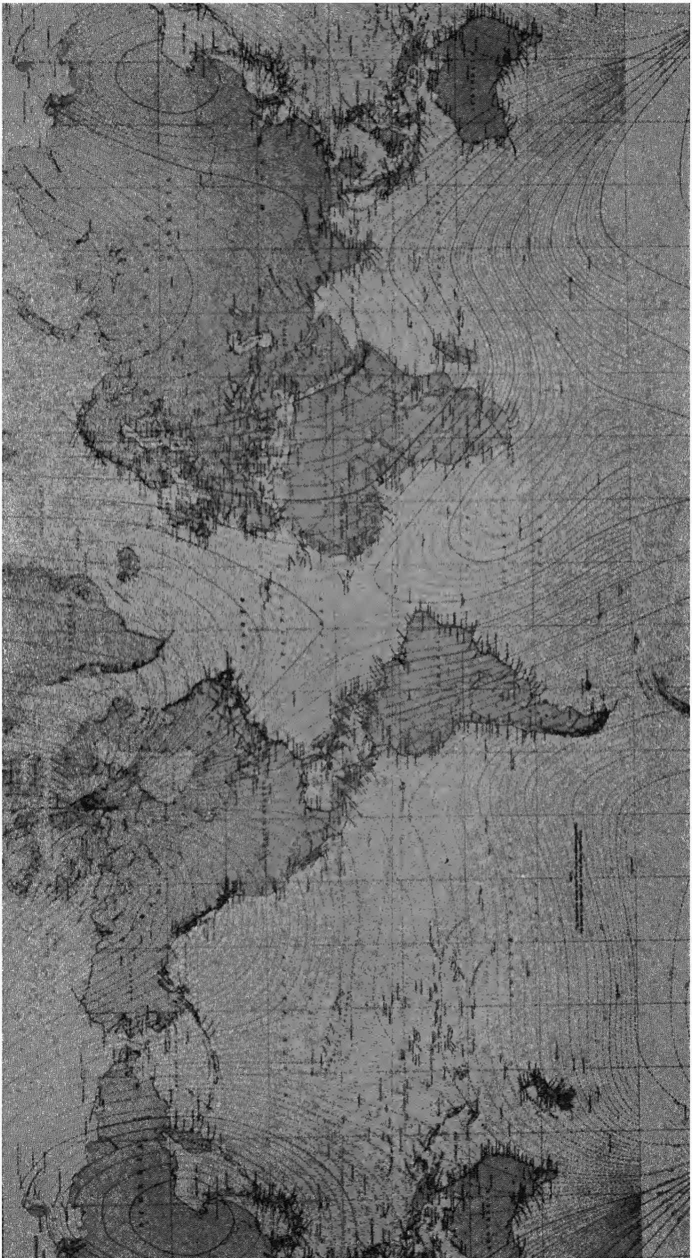

FIG. 138.—Lines of equal declination for 1930.

elements to be discovered and measured. Hellmann[1] says that the first published account of the magnetic declination is found in a book by Falero published in 1535. However, Columbus[2] seems to be credited with the two discoveries: first, that the compass does not always point north; secondly, this deviation from the geographical meridian varies from one place to another. A good surveyor's transit can be used for measuring the declination. This instrument carries both a telescope and a sensi-

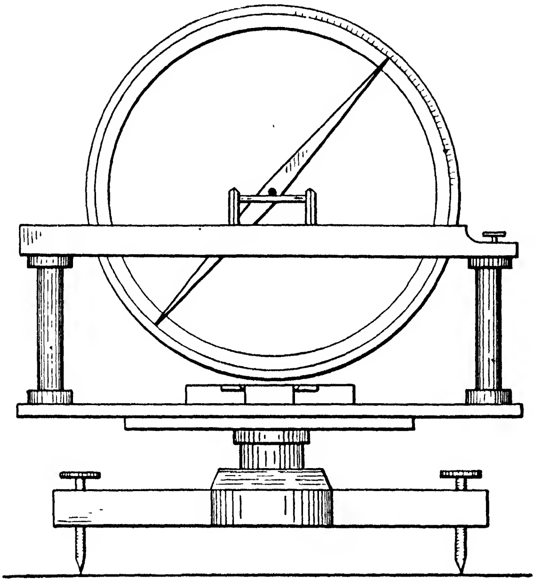

FIG. 139.—A simple form of dipping needle.

tive compass needle. By means of the telescope, pole star observations may be made and the meridian determined. If the transit has been properly constructed, the reading of the compass needle gives the deviation when the axis of the telescope is parallel to the geographical meridian. There are two important factors in building a good transit instrument: first, the needle must rest upon a very accurately ground point, free from burrs; and, secondly, the needle itself must be suspended so near the center of gravity that it quivers as it comes to rest.

If one measures the declination at many points all over the surface of the earth and draws a line through all those places

[1] HELLMANN, *Jour. Terr. Mag.*, 4, 73, 1899.

[2] BAUER, "Principal Facts of the Earth's Magnetism," p. 25, 1914.

which have the same declination, he will get a series of lines like those shown in Fig. 138. These are lines of equal magnetic declination and are called *isogonals.* The continuous lines indicate west magnetic declination and the broken lines east magnetic declination. The three very heavy continuous lines are for those places where there is neither east nor west magnetic declination. The magnetic and geographical meridians coincide at points on these lines which are called *agonic lines.*

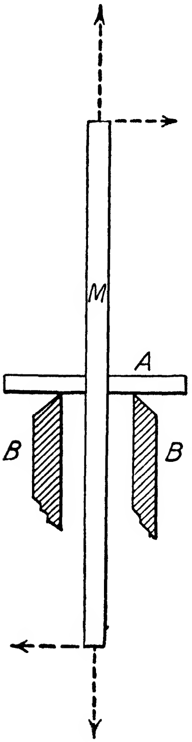

FIG. 140.—A magnetic dip needle *M* is supported by a smooth steel axle *A*, which rolls on the agate knife edges *B* and *B*.

b. Inclination.—The dip of the magnetic needle was discovered by Norman[1] in 1576. The measurement of the angle of dip may be made by a dip needle like that shown in Fig. 139. The needle is very carefully balanced on a fine steel axle which rolls on accurately ground, agate-knife edges (Fig. 140). After it has been carefully adjusted, the needle is magnetized and turned so that the plane in which the needle swings is parallel to the magnetic meridian. The needle will, if set up in the northern hemisphere, dip with the north-seeking pole pointing downward. On a divided circle which surrounds the needle, the angle of dip is measured. After this reading the bearing axle of the needle is turned end for end, and again another reading is taken. This eliminates mechanical dissymmetry. Next the needle is magnetized in the opposite direction and another set of readings taken similar to those mentioned. If the positions of both ends of the needle are observed, this gives a set of eight readings, the mean of which furnishes a fair value of the inclination.

If the values of inclination are measured all over the surface of the earth and lines are drawn through those having equal dip, a series of lines will be obtained similar to those given in Fig. 141. These are called lines of equal inclination, or dip, or just *isoclinal lines.* The line of no dip is the magnetic equator. It will be noted that the isogonal lines do not converge at the geographical poles. In the northern hemisphere

[1] BAUER, "Principal Facts of the Earth's Magnetism," p. 30, 1914.

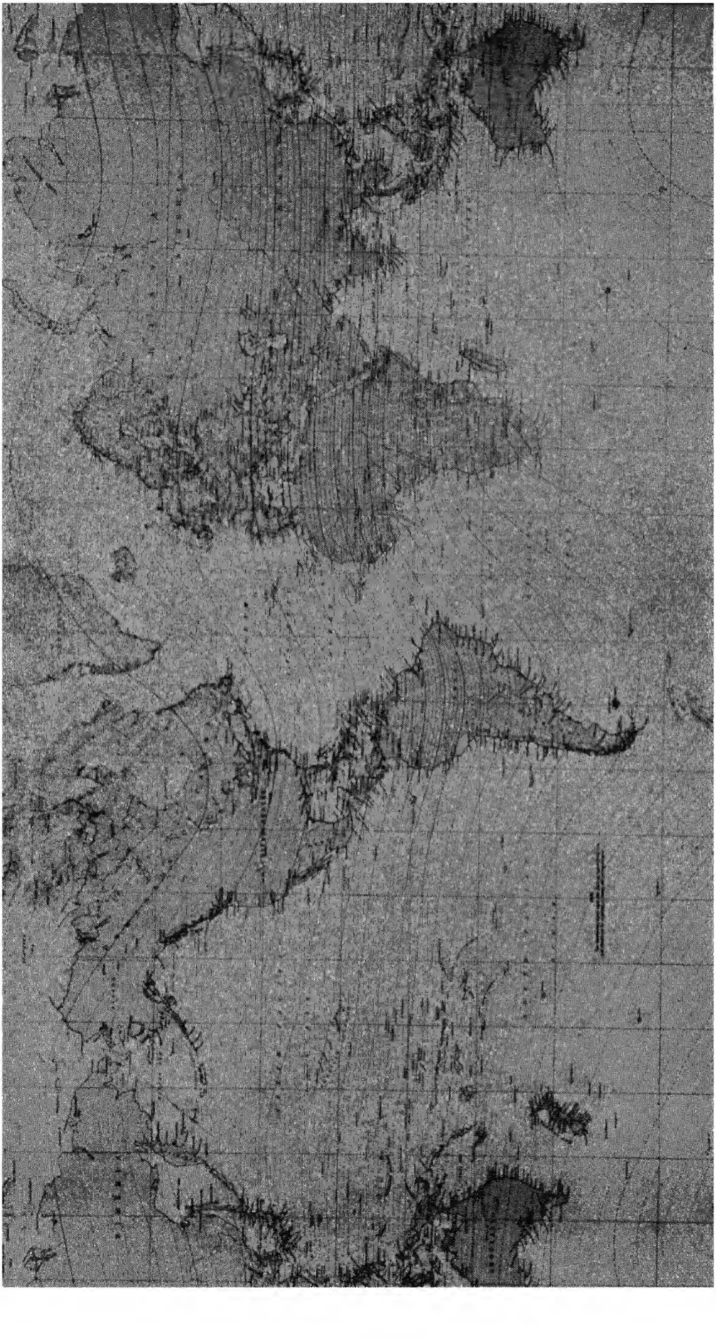

FIG. 141.—Lines of equal inclination for 1930.

they converge at a point where Capt. James C. Ross, in 1831, found there was no directive force acting on the compass needle. This position he found to be 70° 05′ 17″ north latitude and in longitude 96° 45′ 48″ west of Greenwich. At this point the inclination was 89° 59.5′. Ross' location is shown approximately in Fig. 142. This point is called the north magnetic pole. If the earth were uniformly magnetized, a line prolonged through the center of the earth and to the opposite side would

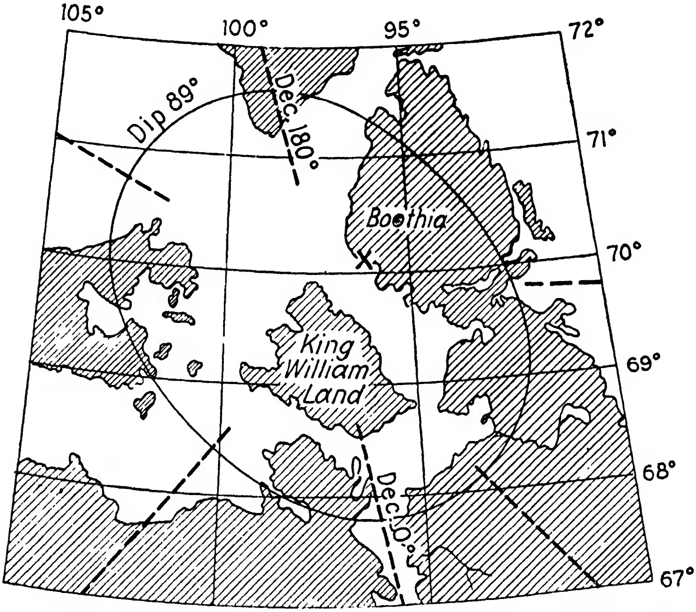

Fig. 142.—The location of the north magnetic pole. (*Schott*, 1890.)

locate the south magnetic pole. The earth is heterogeneously magnetized, so that in order to find the magnetic axis of the earth, the south magnetic pole must be located and the line joining the two will be the magnetic axis. Figure 143 is an attempt to show how, if the earth were permanently magnetized by a large bar magnet inside of it, the axis of the magnet would be turned with respect to the geographical axis. Fleming[1] says, "Taken as a whole, the earth is a feeble magnet. If our globe were wholly made of steel and magnetized as highly as an ordinary steel bar magnet, the magnetic forces at its surface would be at least a

[1] Fleming, *Jour. Terr. Mag.*, **2**, 58, 1897.

hundred times as great as they are now." It seems very probable that the earth's magnetic field is the resultant of a very large number of causes.

c. The Horizontal Component of the Earth's Magnetic Field.—The horizontal component plays a most important rôle among the elements of the earth's field. It is this element of the earth's field which gives direction to the guiding compass. The advent of the modern gyroscopic compass, however, may diminish some of the importance attached to a knowledge of the horizontal component of the earth's magnetic field.

There are several methods for determining the value of H, the horizontal component, but the one most frequently employed is the magnetometric method. Its great advantage is that it is not necessary to use electric currents and, therefore, the apparatus is more portable and not dependent on the accessories of a laboratory. If a small bar magnet is suspended by a fiber in the earth's field, it will vibrate with a period given by the equation,

$$T = 2\pi\sqrt{\frac{K}{MH}}, \tag{189}$$

where K is the moment of inertia of the magnet about the axis of oscillation, M the magnetic moment of the magnet, and H the horizontal component. If this same magnet is brought into the neighborhood of a magnetometer it will cause a deflection of the needle. The deviation of the magnetometer needle will be a function of the magnetic moment of the magnet, the horizontal component of the earth's magnetic field, and the distance between the centers of the magnet and the needle of the magnetometer.

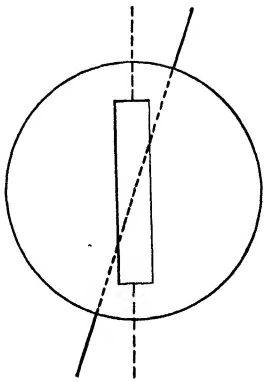

FIG. 143.—If the earth's field were produced by a permanent bar magnet, its axis would be inclined to the earth's axis.

It is a common procedure to set the magnet in a position called "east-west." This is shown in Fig. 144. The magnetometer is arranged so that its needle swings freely in the magnetic meridian. The magnet is turned so that an extension of its axis passes through the center of the magnetometer needle. It was shown (Sec. **10**) that the force due to a magnet acting end-on is

$$F = \frac{2M}{d^3}. \tag{16}$$

If the magnetometer is so arranged that F and H are at right angles to each other there exists the relation,

$$\frac{F}{H} = \tan \Theta, \tag{190}$$

or

$$\frac{2M}{d^3/H} = \tan \Theta.$$

$$\frac{M}{H} = \frac{d^3 \tan \Theta}{2}. \tag{191}$$

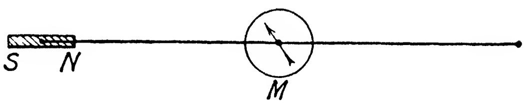

FIG. 144.—Positions of magnetometer and magnet in the method devised by Gauss for measuring the horizontal component of the earth's magnetic field.

Equations (189) and (191) give simultaneous equations for M and H. Solving for H,

$$H = \frac{2\pi}{T} \sqrt{\frac{2K}{d^3 \tan \Theta}}. \tag{192}$$

This simple outline gives only the fundamental part of the experiment, such as would be found in a general physics' laboratory course. The United States Coast and Geodetic Survey issues a booklet entitled, "Directions for Magnetic Measurements," which gives in detail the various corrections which should be made for accurate work. Other methods[1] involving electric currents depend for the most part on balancing a known magnetic field against the unknown field of the earth.

The *earth inductor* is another important piece of apparatus for measuring both the horizontal and vertical components of the earth's magnetic field. It consists of a coil C which can be rotated quickly (Fig. 145), either about a horizontal or vertical axis. It is arranged with a spring, so that when the coil is released, it will swing through 180°. The earth inductor is connected in a circuit comprising a variable resistance and a ballistic galvanometer whose constant is known. From the effective area of the coil, the resistance of the circuit and the constant of the galvanometer, the strength of the magnetic field, in which the coil is rotated, may be determined. When the plane of the

[1] SCHUSTER, *Jour. Terr. Mag. Atmos. Elec.*, **19**, 19, 1914; WILLIAMS, *Phys. Rev.*, **22**, 204, 1923.

coil is normal to the horizontal component of the earth's magnetic field, a rotation of 180° gives a measure of the horizontal component. When the coil is normal to the vertical component, a similar rotation of 180° gives the vertical component. The theory of the earth inductor is very simple. The coil, by its rotation in the earth's field, varies the number of lines of force threading through it. This produces an electromotive force in the coil,

$$\text{Emf} = -\frac{dN}{dt}. \tag{1}$$

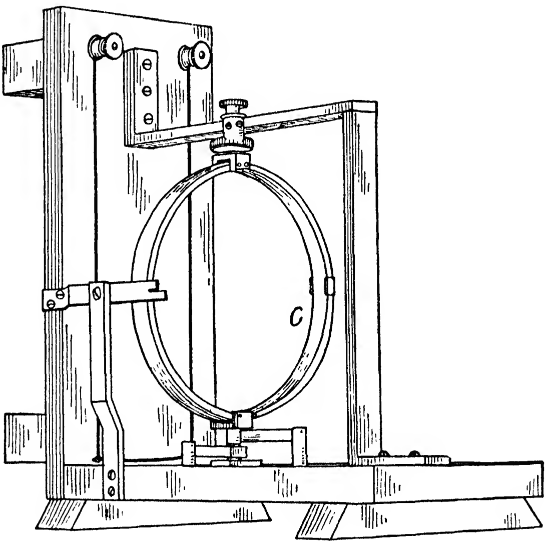

FIG. 145.—A laboratory form of earth inductor.

The number of lines of force N, threading through the coil at any instant, will be

$$N = HA \sin \theta, \tag{193}$$

where H is the field being measured, A the effective area of the coil, and Θ the angle which the plane of the coil makes with the direction of the field.

$$\text{Emf} = -\frac{dN}{dt} = -\frac{d(HA \sin \Theta)}{dt} \tag{194}$$

$$I = -\frac{HA}{R} d(\sin \Theta)\cdot$$

R is the resistance of the coil and galvanometer plus any variable resistance which may be in the circuit. If at any time $t = 0$, $\Theta = \pi/2$ and at an instant later, $t = t_1$, $\Theta = -\pi/2$, then

$$\int_0^t I dt = Q = -\frac{HA}{R}\int_{\Theta=\frac{\pi}{2}}^{\Theta=-\frac{\pi}{2}} d(\sin\Theta)$$

$$= -\frac{HA}{R}\Big[\sin\Theta\Big]_{\frac{\pi}{2}}^{-\frac{\pi}{2}} = \frac{2HA}{R}.$$

In terms of the deflection of the ballistic galvanometer,

$$Q = \frac{2HA}{R} = k\phi \tag{195}$$

$$H = \frac{Rk\phi}{2A}. \tag{196}$$

k is the constant of the ballistic galvanometer, and ϕ is its deflection. For the vertical component of the earth's field, Eq. (196) becomes

$$V = \frac{Rk\phi_V}{2A},$$

while for the horizontal component there follows,

$$H = \frac{Rk\phi_H}{2A}.$$

The tangent of the angle of dip is,

$$\tan\text{(dip)} = \frac{V}{H} = \frac{\phi_V}{\phi_H}. \tag{197}$$

This is one of the standard methods for getting the components of the earth's magnetic field.

d. The Vertical Component of the Earth's Magnetic Field.—The earth inductor has just been shown to be applicable to determination of both the horizontal and vertical components. If, by the magnetometric method, H can be determined and the dip evaluated from the dip needle, then by Eq. (197) V can be calculated. These two methods are the commonest. In speaking of the various observations to determine the elements of the earth's field little or nothing has been said about corrections for various errors. Detailed discussion of these corrections are given in the booklet issued by the United States Coast and Geodetic Survey,[1] and in various papers published by the Depart-

[1] *Loc. cit.*; also BAUER, "Principal Facts of the Earth's Magnetism," p. 79, 1919.

ment of Terrestrial Magnetism of the Carnegie Institution of Washington.[1]

68. Variations in the Magnitude of the Earth's Magnetic Elements.—The various elements which have just been discussed are continually varying. Some of the variations are periodic and others are sudden and irregular. There are five variations which are commonly cited:

a. Diurnal or daily.
b. Annual.
c. Secular.
d. Magnetic storms.
e. Minor periodic variations.

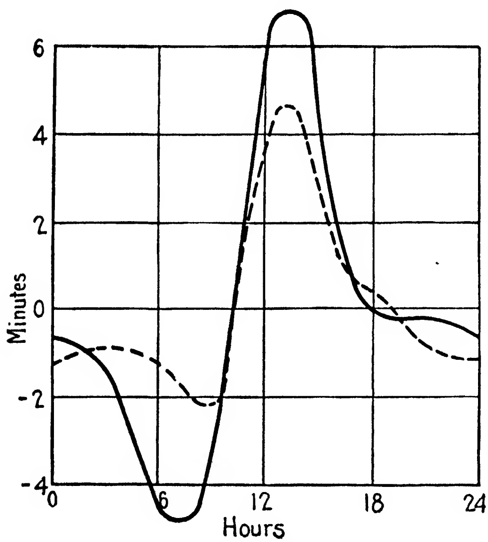

FIG. 146.—The average diurnal variation of declination at the Kew Observatory, England. Continuous line is for summer; dotted line, winter.

a. Diurnal or Daily Variations.—By means of recording magnetometers[2] of various types it is possible to follow the daily changes in the declination, horizontal intensity, inclination, and vertical intensity of the earth's field. Figure 146 gives the curves showing the summer and winter diurnal variation of declination at the Kew observatory, England. These curves vary from place to place on the earth's surface. "Principal

[1] *Journal of Terrestrial Magnetism.*
[2] WATSON, *Jour. Terr. Mag.*, **6**, 187, 1901;
ESCHENHAGEN, *Jour. Terr. Mag.*, **5**, 59, 1900.

Facts of the Earth's Magnetism," by Bauer, gives some very interesting magnetograms of the daily variations as recorded at the Cheltenham Magnetic Observatory with the Adie magnetograph. The daily variation is not the same day after day. There are so-called quiet days and days of considerable disturbance. Chree[1] has made a study of the differences in variation between quiet and disturbed days. The values which he used in showing the differences in declination between the two types of days were the means over an 11-year period. Schuster[2] came to the conclusion, from his investigations of the daily variations, that the cause for it must lie outside of the earth, *i.e.*, the probability seems to point to electric currents in the atmosphere. The flow of atmospheric electricity and a study of its behavior is exceedingly important in connection with a study of the variations in the earth's magnetic field.

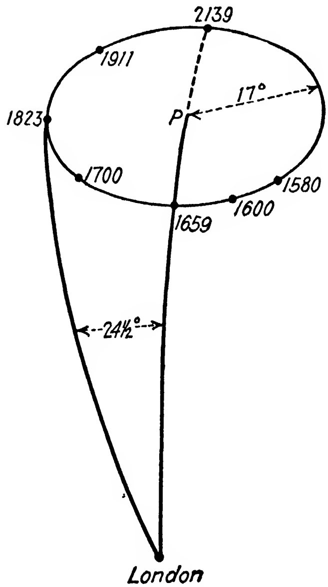

FIG. 147.—Graphical representation of the secular variation of the declination as observed at London.

b. Annual Variations.—The annual variation is small in comparison with the daily. It is not to be confused with the variations which occur from one year to another due to one of the other changes. So far as the annual variation is concerned the magnitude of the variable at the end of the year is virtually the same as at the beginning. In the case of the magnetic declination the variation may be derived by taking the declination month by month and correcting for any of the other variations which would have an influence on it.

c. Secular Variations.—These are the most pronounced of all and occur over long periods of time. The main secular variation seems to be cyclic. Gellibrand in 1634 discovered and recorded the secular variation of the declination of the earth's field. The cause of this, like the origin of the earth's field, is still a matter of conjecture. Not only the declination, but

[1] CHREE, *Proc. Roy. Soc.*, **91**, 370, 1914–1915.

[2] SCHUSTER, *Philos. Trans.*, **180**, 467, 1889.

also the dip and intensity undergo these long time changes. Lord Kelvin pointed out that the magnetic pole is slowly moving from east to west and while observations do not go back far enough to make it exact, it appears that the period is about 960 years. The north magnetic pole describes a small circle on the earth's surface of about 17° radius. Figure 147 shows how this movement of the magnetic pole influences the declination of a place like London. The earliest of these records dates from 1580. Bauer,[1] however, pointed out that a study of the declination alone

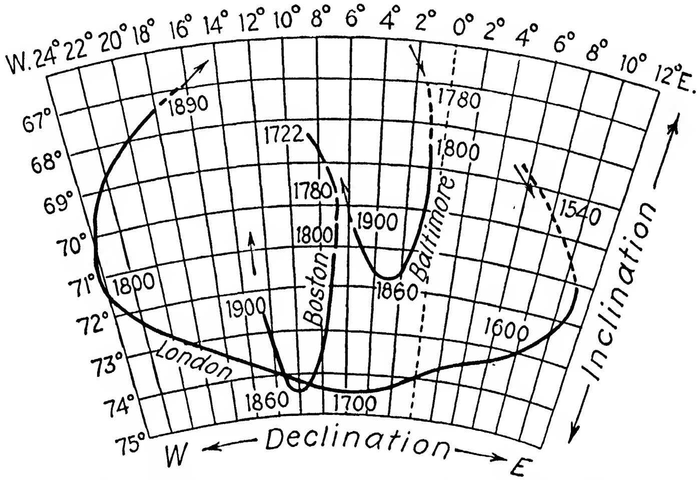

FIG. 148.—Curves showing secular change in magnetic declination and dip at London, Boston, and Baltimore.

would not tell the story completely. What one should ask is, "How does the north end of the freely suspended magnetic needle move with the lapse of time, if the motion is observed from the point of suspension of the needle?" If the north-seeking pole of a freely suspended needle follows the movement of the earth's magnetic pole, it will describe in space a curve like that shown in Fig. 148. Bauer has worked out these curves for Baltimore, Boston, and London, and in all cases the direction of rotation of the needle point is clockwise. It will be seen in Fig. 148 that both the declination and the dip variations have been visualized. There are a few stations on the western coast of the United States which exhibit anti-clockwise motion over short periods. This movement of the earth's field over the surface of the globe seems

[1] BAUER, "Principal Facts of the Earth's Magnetism," p. 45, 1919.

to preclude the magnetic properties of the earth's crust from playing an important part in fixing the magnitude of the field.

d. Magnetic Storms.—These magnetic disturbances are characterized by their sudden advent. The instruments, which photographically record the declination, and vertical and horizontal intensities, show fairly sharp breaks[1] in the curves on the registration sheets when these spasmodic variations occur. Bauer[2] says:

> There are clearly three kinds of magnetic storms: (1) Cosmic ones, due to changes occurring in the regions above; (2) telluric ones, resulting from changes within the interior of the earth; and (3) regional or local ones, resulting from changes within or external to the earth's crust, whose field of action is limited to a restricted region of the earth and the center or focus of which, while sometimes stationary, generally travels from place to place.

The principal phases of a storm of the first kind appear to occur simultaneously over the whole earth. Telluric currents are very manifest during these disturbances. Bauer points out that,

> A magnetic storm of the second category is associated with changes within the earth, cataclysms, earthquakes, volcanic outbreaks, etc. The phases may occur simultaneously over very large portions of the earth, or progress from place to place according to a certain rate. Remarkable coincident effects were observed during the May eruption in Martinique.

The third type of magnetic storms takes place over a limited area. Not infrequently they appear as secondary phenomena accompanying the first two kinds of storms. There are times when the earth's currents, which occur in connection with magnetic storms, have such a magnitude that they have been known to throw telephone instruments out of commission. Brilliant auroral displays are a very frequent concomitant phenomena. Rodes[3] has recently given a very interesting paper on the cause and propagation of magnetic storms.

e. Minor Periodic Variations.—There are solar influences over and above the variations which have already been mentioned. Nippoldt gives Kreil the credit for having discovered that the

[1] NIPPOLDT, "Erdmagnetismus, Erdstrom und Polarlicht," p. 71, 1921.
[2] BAUER, "Principal Facts of the Earth's Magnetism," p. 55, 1919.
[3] RODES, *Jour. Terr. Mag. Atmos. Elec.*, **32**, 127, 1927.

moon also plays a part in the variations observed. Observations seem to indicate that the planets come into the calculations also. Lightning, earthquakes, and eclipses[1] of the sun come under the designation of minor influences. A perusal of the leading textbooks and papers dealing with these five variations indicates that there are no hard and fast lines separating one kind of variation from another. This must follow from the fact that there are so many theories regarding the causes of the variations.

69. Eleven-year Periods of Disturbances.—From a study of some of the variations which have just been discussed it emerges that there is an ebb and flow in their values which find maxima about every 11 years. Records have been kept of the occurrence of magnetic storms from year to year and it has been observed that a maximum number occurs about every 11th year. A similar periodicity has been observed in the appearance of the northern lights. Both of these phenomena synchronize with the number of sun spots which have been recorded year by year. This 11-year period seems to be a very important one. Even economists see in this 11-year cycle of sun spots a reason for a similar periodicity of prosperity which they think they have discovered. It is even asserted that the rings of a tree show the cyclic arrangement, the maximum number of thick rings coming every 11 years.

70. Theories Concerning the Earth's Magnetism.—The first great advance in the theory of the earth's magnetism was made by Gauss[2] in 1839. In this work he developed a formula which should represent the earth's field at any point on the surface of the earth, The agreement, which Gauss found, between the observed and calculated values, confirmed him in the assumption that the earth's field is mostly due to internal magnetism. Gauss suggested that the variations in the magnetic elements might be considered in a fashion similar to that in which he had treated the main field. This was undertaken by Schuster[3] in 1870, and from his theoretical study he came to the conclusion that the daily variation is due to causes outside the earth and suggested electric currents in the atmosphere. The daily varia-

[1] BAUER, *Jour. Terr. Mag. Atmos. Elec.*, **15**, 2, 1910.

[2] GAUSS, "Papers," vol. V, p. 119;
POYNTING and THOMSON, "Electricity and Magnetism," p. 316, 1920.

[3] SCHUSTER, *Philos. Trans.*, **180**, 467, 1889.

tion of these aerial currents induce electric currents in the earth which affect the amplitude of the vertical and horizontal components.

More recently Nippoldt[1] has given a physical theory of the earth's magnetic field. A summary of his paper is given as follows:

The principal part of the earth's magnetic field consists of a non-homogeneous magnetization of the earth's crust down to a depth of about 20 km. In addition, there may exist, for the earth's nucleus, a magnetic field symmetrical both about the earth's axis of rotation and the equatorial plane, which may have arisen in the same manner as has the sun's general magnetic field. Besides these two fields there may be, in general, a third, interplanetary field, whose axis is perpendicular to the plane of the ecliptic and which may be caused by the sun's electric radiation.

The Department of Terrestrial Magnetism of the Carnegie Institution of Washington is now engaged in an extensive survey of the earth's electric currents, the electric flux in the atmosphere, and the variations of the magnetic field of the earth in the hope of further correlations. This is highly important. It is essential that there be an accumulation of a great deal of data extending over a fairly long series of years before the real fruit of their labors may be realized. While there is much to learn concerning the magnetic field of the earth, it suggests that there may be as much or more to be learned about all the heavenly bodies.

71. Stellar Magnetism.—Until the year 1908 it was a matter of conjecture as to whether the sun possessed a magnetic field or not. In that year Hale[2] was able to demonstrate, beyond the shadow of a doubt, that the sun did possess a magnetic field, even though our conception of a white-hot body possessing magnetism was against such an idea. The phenomenon discovered by Zeemann has already been considered (Sec. **64**). It was the discovery of this effect in the spectrum of the sun spots by Hale which proved the existence of a magnetic field there, of sufficient power to resolve the light from the sun spots into its various components. The separation of the spectral lines indicated that the field intensity necessary for such a resolution as Hale found should be in the neighborhood of 50,000 gausses.

[1] Nippoldt, *Jour. Terr. Mag. Atmos. Elec.*, **26**, 99, 1921.

[2] Hale, *Astrophys. Jour.*, **47**, 235, 1918.

This is a field strength produced only by the best of electromagnets as they are generally constructed. Hale found that the sun spots were whirling storms[1] in which the spots were the centers about which great masses of electrically charged particles revolved and gave a magnetic field, just as a coil of wire would. These fields then acted upon the light which came to the spectroscope from that part of the sun spot being observed. In discovering the magnetic field of the sun spots, the conditions were favorable because it was quite possible to get a sun spot in such a position that one was looking into it or along its magnetic axis.

Later, Hale was able to show that the sun as a whole possessed a magnetic field, the axis of which coincided with the axis of rotation. This was a much more difficult undertaking than the previous discovery. The total field of the sun is very much weaker than that found in the local sun spots, being only about 25 gausses. This is about 40 times that of the total intensity of the earth's magnetic field. The magnetic field of the sun as a whole is too weak to have its influence felt here on the earth. It is only through the influence of the sun's magnetic field on the light emitted by the sun that we become aware of that field. The diurnal and annual variations in the earth's magnetic field are not to be accounted for by the field of the sun. Even in the case of magnetic storms, it is not the magnetic field of the sun as a whole or of its spots which is setting these things off, but, as Hale indicates, probably enormous streams of electrons which are shot out from the spots and penetrating our atmosphere give rise inductively to some of the effects we observe in aerial and telluric currents.

If the earth and the sun possess magnetic fields, it is an easy flight of the imagination to suppose that the stars and the planets may also have theirs. To know more of these magnetic fields which throw their subtle influence far out into space is an absorbing program of research which the epochal work of Hale has inaugurated.

[1] HALE, "The New Heavens," p. 64, 1922.

CHAPTER VIII

MAGNETIC THEORIES AND SOME EXPERIMENTAL FACTS

72. Early Observers and Theorists.—In a *Report* issued by the National Research Council on "Theories of Magnetism," the first chapter is devoted to a résumé of the outstanding theories which have promoted our knowledge of magnetic phenomena. It is not the purpose of this book to dwell on theory, but rather to emphasize the experimental. However, they must go hand in hand to a further and ultimate understanding of magnetic phenomena, just as experiment and theory have supplemented each other in the way we have already come.

Gilbert's "De Magnete"[1] (1600) opened up the experimental method even though enshrouded too often in speculations. Nearly 200 years later with the advent of better experimental methods, Coulomb established his laws of magnetic attraction and repulsion (Sec. 4) which Poisson[2] used as a foundation for his theory Poisson pictured magnetism as a two-fluid affair, and the process of magnetization as a separation of the two. This fitted in with the phenomena of static electricity, which seemed most readily explained in this manner.

Oersted's discovery, in 1820, gave a great impetus to the study of magnetic phenomena. It led Ampère to the theory of molecular currents, a theory which supposed minute electric currents flowing in the molecules. This theory further stipulated that the fields of the molecules were turned parallel to the magnetizing force when the process of magnetization ensued. Then came Faraday's[3] researches which gave the whole field of magnetism a new stimulus and which Weber[4] built into the hypothesis of Ampère. Weber's theory failed to account for residual magnetism. Maxwell[5] modified Weber's theory in order to correct this lack.

[1] Gilbert, "De Magnete," translated by the Gilbert Club.

[2] Poisson, "Sur la Théorie du Magnétisme," *Mém. de l'Inst.*, **5**, 247, 488.

[3] Faraday, "Experimental Researches," vol. I, p. 2.

[4] Weber, *Poggendorff Ann.*, **87**, 145, 1852.

[5] Maxwell, "Electricity and Magnetism," vol. II, sec. 444.

Again, in the latter half of the nineteenth century, experiment once more stimulated theory. Rowland's[1] work showed that theory was not adequate to explain hysteresis. Ewing,[2] in order to explain this phenomenon and residual magnetism, introduced the idea of mutual constraint which the molecules exert on each other. By means of a model, he demonstrated how this would account for hysteresis and also residual magnetism. A later study[3] of the stability of groups of small magnets, which Ewing used in the first model, led him to abandon the old model. In the new model each atom forms a magnetic system comprising a Weber element capable of turning, and controlled by the magnetic forces acting upon it. While this model has advantages in producing the desired stability, Honda[4] feels that the "new model does not accord with the observed facts." This last magnetic model is, of course, largely influenced by our modern concepts of atomic structure. All the theories which preceded it were not from the modern electron theory of matter, which conceives each atom of matter as made of a central nucleus about which electrons are revolving. The modern electrical theory of matter gives some reality to the Weber element.

73. Rutherford-Bohr-Sommerfeld Atom.—Rutherford,[5] from his experimental work on the scattering of α-particles, was led to postulate the atom as made up of a nucleus of very small dimensions and yet possessing most of the mass of the atom. About the nucleus revolved, like satellites, the electrons. The number of these orbital electrons depended upon the position of the atom in the series of elements. Hydrogen, the lightest element, would have 1 electron, helium 2, lithium 3, and so on up to uranium with 92. More characteristic than atomic weight would be the atomic number. The latter would indicate the number of orbital electrons and the position of the element in the series of 92. Based on the work of Rutherford, Bohr[6] built up a very helpful theory of radiation to which Sommerfeld,[7] invoking the principle of relativity, has added a theory for the fine line structure in

[1] ROWLAND, *Philos. Mag.*, **46**, 140, 1873; **48**, 321, 1874.

[2] EWING, "Magnetic Induction in Iron and Other Metals," p. 287; HONDA and OKUBO, *Sci. Repts. Tóhoku Imp. Univ.*, **5**, 153, 325, 1916.

[3] EWING, *Proc. Roy. Soc. Edinburgh*, **42**, 97, 1922.

[4] HONDA and OKUBO, *Sci. Repts. Tóhoku Imp. Univ.*, **12**, 27, 1923.

[5] RUTHERFORD, *Proc. Roy. Soc.*, **97**, 374, 1920.

[6] BOHR, *Philos. Mag.*, **26**, 467, 857, 1913.

[7] SOMMERFELD, "Atombau," 4th ed., chap. VIII.

spectroscopy. While magneticians as a whole would not agree to saying that a comprehensive theory of magnetism has been evolved, yet, in all probability, they would agree that the final theory of magnetism must start from a foundation which involves the major postulates of the Rutherford-Bohr-Sommerfeld atom.

The mathematical theory of magnetic induction as worked out by Poisson, Green, and Kelvin is fairly complete, because it is statistical and, like thermodynamics, needs no model or picture on which to build. Our modern theories are leading us to a study of the dynamics of the ultimate magnetic particle. There is a demand to know what is going on in that unit we call the elementary magnet. Is there a distinct movement of this unit when acted upon by a magnetizing force while surrounded by an infinite number of its own kind? From the discussion of the various magnetic phenomena which have been given in the preceding chapters, it is evident that a theory of magnetism which will explain all of them will have to be a very comprehensive one. A theory which will explain only a few and no others is a theory that must eventually be discarded. One has only to consider the varied phenomena of magnetostriction to realize how much needs to be done in developing the desired theory.

74. Curie-Langevin-Weiss Theories.—The beautiful work of Curie has already been discussed (Sec. **35**e). Curie's work is another instance where the experimental results have stimulated further theoretical investigations. Out of Curie's work grew a belief that the susceptibility of paramagnetic bodies was inversely proportional to the absolute temperature. So many exceptions to this idea have been found, however, that it can no longer be taken as a generalization.

With Curie's work as a background, Langevin[1] considered the conditions which would arise if a magnetic field were applied to a paramagnetic gas like oxygen. That there would be a rotation of the elementary magnets was to be expected, but that there should arise also a loss of potential energy and, therefore, an increase in temperature was not so apparent. To this condition Langevin applied the laws of thermodynamics and obtained the equation:

$$K = \frac{M^2N}{3RT}, \tag{198}$$

[1] LANGEVIN, *Ann. chim. phys.*, **5**, 70, 1908.

which is another statement of Curie's law that $K \propto 1/T$, where K is the susceptibility, M the magnetic moment of the gas molecules, and N the number of molecules per cubic centimeter.

The chapter on magneto-thermics tried to make it plain that temperature has a most profound influence on magnetic properties. At 310° C., for instance, nickel loses all of its ferromagnetic properties. Langevin's theoretical considerations tried to bring into the picture of magnetic processes the effects of temperature. Unfortunately, his theory does not include ferromagnetism. Weiss,[1] therefore, attempted to enlarge Langevin's theory. In order to account for the great intensity of magnetization in ferromagnetic bodies, Weiss added an intrinsic field which should be proportional to the intensity of magnetization. This internal field acted in the same direction as the external force and was due to the action of neighboring molecules.

From Eq. (198) it is possible to calculate M, which Weiss found to be a multiple of a definite number for each element. This seemed to indicate that there was an elementary magnetic moment just as in electrical phenomena there is an elementary electrical charge which we call the electron. Weiss compared the values of M as deduced from the saturation intensities at absolute zero. His results showed that the greatest common divisor of the values of M for iron and nickel is 1.85×10^{-21} e.m.u. The magnetic moment of iron is 11 times greater than this unit while nickel possesses a magnetic moment of $(1.85 \times 10^{-21})3$. A great deal of controversial writing has appeared concerning the significance of the Weiss magneton and the Langevin-Weiss theory. Bohr has arrived at a magneton approximately five times that of Weiss, $M = 9.2 \times 10^{-21}$ e.m.u. While there are a great many facts[2] which do not fit into the theory proposed by Langevin and Weiss, nevertheless it has made a very great contribution towards a better understanding of the relation between dia-, para-, and ferromagnetism.

75. Other Theories of Magnetism.—A number of modifications of the Langevin-Weiss theory have been proposed. Gauss[3] considered the elementary molecular magnet as being charged

[1] WEISS, *Jour. de phys.*, **6**, 661, 1907; *Arch. sci. phys. et nat.*, **31**, 401, 1911.

[2] STRADLING, *Jour. Franklin Inst.*, **180**, 173, 1915;

DUSHMAN, "Theories of Magnetism," *Gen. Elec. Rev.*, May, August, September, October, December, 1916.

[3] GAUSS, *Gött. Nachr.*, p. 197, 1910; p. 118, 1911.

with electricity and revolving rapidly about its axis. He was able to get a very good expression for the hysteresis curve in ferromagnetic crystals, which qualitatively, at least, agreed with observations. Schrödinger[1] considered the free electrons as a cause of diamagnetism. A statistical theory by Jan Kroo[2] gives an explanation for diamagnetic polarity but not for para- and ferromagnetic bodies. Honda[3] claims some excellent agreements between his observations and calculations in a theory of magnetism based on the kinetic theory of Langevin.

Darrow[4] makes an interesting comparison between Ewing's theory and the one proposed by Langevin and Weiss. The two points of view seem contradictory. In Ewing's theory,

> the perpetual effort of the applied field to align the elementary magnets is hindered by the forces which these exert on each other. In Langevin's theory the antagonist of the applied field is the thermal agitation. Now Langevin's theory is competent to deal with paramagnetic substances which are difficult to magnetize, but not with iron and the like which are strongly magnetized by weak fields. This means that the thermal agitation is too strong an antagonist to the applied field. Weiss, therefore, provided the latter with a powerful ally in the form of an intense molecular field parallel to it and proportional to the magnetization. The applied field and its ally together are able to overpower the thermal agitation and bring about saturation in cold iron.

Out of this apparent contradiction, Darrow raises the question if, after all, these two opposing conditions may not be complementary to each other. There seem to be some possibilities along this line of procedure. Both theories have made very great contributions to our knowledge of magnetic processes. Features of both theories seem worthy of retention in an ultimate theory.

The past 25 years have seen a great deal of experimental work done in the field of magnetostriction. The investigations of others along with his splendid work on the magnetostriction of permalloy and other metals have led McKeehan[5] to a theory of ferromagnetism in which magnetostriction plays the leading rôle. The distortion which occurs in the Joule effect is the principal cause of hysteresis and for the gradual rather than sudden rise of the *I-vs-H* curve. He was led to this point of

[1] Schrödinger, *Sitz.-ber. Akad.*, Wien, **121**, 1305, 1912.

[2] Kroo, *Ann. der Phys.*, **42**, 1354, 1913.

[3] Honda, *Sci. Repts. Tôhoku Imp. Univ.*, **3**, 173, 1914; **7**, 141, 1918.

[4] Darrow, *Bell Syst. Tech. Jour.*, **6**, 361, 1927.

[5] McKeehan, *Phys. Rev.*, **26**, 274, 1925; **28**, 158, 1926.

view in studying the magnetic properties of nickel-iron alloys, in which at a certain per cent of nickel the Joule effect and hysteresis became zero and yet the initial permeability was very great. Schulze,[1] in an extensive study of the magnetostriction of various metals and alloys, takes exception to McKeehan's theory.

In a recent paper by Honda[2] a theory of magnetism, based on the structure of the atom, is proposed. He considers the nuclear electrons as being the cause of para- and ferromagnetism, while diamagnetism is attributed to the outer or optical electrons. While Honda[3] does not favor Ewing's new model of the elementary magnet, yet his recent theory seems to lend itself to Ewing's picture because the nuclear magneton furnishes the Weber element in Ewing's model.

76. The Quantum Theory.—The quantum theory has had a very useful existence in the field of spectroscopy. That it may also be of great use in studying magnetic phenomena seems to be confirmed by the experiments of Gerlach and Stern.[4] They shot a narrow stream of silver atoms through an intense heterogeneous magnetic field in such a direction that the atoms moved normal to the field and its gradient. The theory of spatial quantization[5] stipulates that an atom, which possesses a resultant moment of momentum, cannot take up all directions of orientation in a magnetic field, but only those which are an exact multiple of $he/4\pi m$. In the case of the Bohr magneton, its alignment must be either parallel to the field or oppositely directed. Consequently, the atoms whose axes are parallel to the field will be urged in one direction and those turned oppositely will be pushed the other way. On the quantum hypothesis, the beam of silver atoms should be split in two, while on the classical it should simply be broadened. The experiment, which Gerlach and Stern performed, showed a splitting of the beam, thus giving a decision in favor of the quantum theory. A direct estimate of the magnetic moment of the silver atom was in favor of the Bohr magneton. The work of Gerlach and Stern is the "most direct proof of

[1] Schulze, *Zeitsch. für Phys.*, **50**, 483, 1928.

[2] Honda, *Proc. Imp. Acad.*, **4**, 12, 1928.

[3] Honda, *Sci. Repts. Tóhoku Imp. Univ.*, **12**, 27, 1923.

[4] Gerlach and Stern, *Ann. der Phys.*, **74**, 673, 1924; **76**, 163, 1925.

[5] Sommerfeld, *Physik. Zeitsch.*, **17**, 491, 1916;
Debye, *Gött. Nachr.*, June, 1916.

the quantization of the direction of the moment of the atom in space with respect to the direction of the magnetic field."[1]

Einstein and Ehrenfest[2] have shown that there are difficulties in the way of explaining the Gerlach-Stern experiments, both from the classical and the quantum theories. Nevertheless, this beautiful and difficult experiment is destined to become classical because of its bearing on magnetic theories, for here one is dealing with the direct effects of a magnetic field on the atoms of an element.

77. Theory of Rotation of Elementary Magnets.—Almost without exception every important theory of magnetism assumes the idea of something rotating within the medium when it is subjected to a magnetic field. Is that element, which rotates, a small aggregate (crystal) of atoms or the atoms themselves? Could the nucleus or the orbital electrons of the atoms form the rotating part? This point is not specifically stated in any of the theories. What is the evidence for and against something turning over within the medium when magnetized?

It is, of course, an old observation that a magnet seeks to set its axis parallel to a magnetic field. The magnetization of a piece of iron when placed in a magnetic field was very easily explained by saying that elementary magnets were rotated and aligned themselves parallel to the imposed field. It was an obvious conclusion to draw that, in the case of permanent magnets, these elementary magnets remained oriented more or less in the same direction. The experiments of Baily[3] fit into this picture also. Swinburn had pointed out that Ewing's model, if it were correct, would indicate that a piece of iron rotating in a magnetic field would have a falling off in the amount of hysteresis as the point of saturation was reached. This was confirmed by Baily and would indicate that the elementary magnets stay oriented in the strong field even though the mass as a whole rotated.

The careful and painstaking work of Barnett[4] on the production of magnetization by rotation seems to be another confirmation of something rotating within the medium. If the elementary magnets are electrons revolving in orbits about the

[1] Andrade, "The Structure of the Atom," p. 619, 1927.

[2] Einstein and Ehrenfest, *Zeitsch. für Phys.*, **11**, 31, 1922.

[3] Baily, *Philos. Trans.*, **187**, 715, 1896.

[4] Barnett, *Proc. Amer. Acad.*, **60**, 127, 1925.

atomic nuclei, they will possess angular momenta. Simultaneously, they will have a resultant magnetic moment. If a rod, made up of these magnetic carriers, is rotated, the gyromagnetic atoms will have their angular momenta changed. The resultant magnetic moment will also be changed, which is magnetization by rotation.

The converse of this experiment is one suggested by Richardson[1] and experimentally confirmed by Einstein and DeHaas,[2] Chattock and Bates,[3] Beck,[4] and Stewart.[5] A slim cylindrical rod is supported by a fine fiber which is a continuation of the axis of the rod. A sudden longitudinal magnetization of the rod changes the angular momenta of the magnetic carriers and, therefore, of the structure of the rod. This will be manifested by a ballistic throw of the rod as a whole. There is a need to study diamagnetic bodies and see if any such effect is present there.

Another bit of evidence pointing toward the idea of rotating elementary magnets is to be found in the Barkhausen effect. An oscillograph of the Barkhausen effect was shown in Fig. 108. If one thought of a group of the elementary magnets in a piece of iron as flopping over in an irregular fashion, sometimes more in one group than in the other, then one might expect the electromotive force shown in Fig. 108. This can actually be carried out as a model by rotating a group of small broken pieces of magnetite in a tube inside of a coil connected to the input side of an amplifier. In this experiment there are groups of small magnets turning first in one direction and then in another. At any given instant there is a resultant, due to the algebraic sum of the effects of all the little magnets. When the tube containing the magnetite is rotated fast enough there is not any marked difference between Figs. 108 and 149. The foregoing explanation of the Barkhausen effect is supported by Forrer's[6] experiments in which he finds discontinuity of phenomena in the magnetization curve of nickel.

The old model of Ewing's has doubtless had a very profound influence on our thinking about magnetic phenomena. Darrow[7]

[1] RICHARDSON, *Phys. Rev.*, **26**, 248, 1908.

[2] EINSTEIN and DEHAAS, *Verh. d. Deutsch. phys. Gesellsch.*, **17**, 152, 1915; **18**, 173, 1916.

[3] CHATTOCK and BATES, *Philos. Trans.*, **223**, 257, 1922.

[4] BECK, *Physikal. Zeitsch.*, **20**, 490, 1919.

[5] STEWART, *Phys. Rev.*, **11**, 100, 1918.

[6] FORRER, *Jour. phys. et le radium*, **7**, 109, 1926.

[7] DARROW, *Bell Syst. Tech. Jour.*, **6**, 343, 1927.

thinks, "So great a result is attained from so simple an apparatus, [Ewing's model] that it seems very unlikely that any radically different explanation of either quality [gradual magnetization and hysteresis] will ever be put forth." In a large group of small magnets, carefully pivoted so as to turn about vertical axes, Ewing demonstrated that, as these model elementary magnets turned, he was able to measure the *I-vs-H* relations and also determine the hysteresis for such a group. One saw the magnets turn in the process. It was a model which could be demonstrated to the eye.

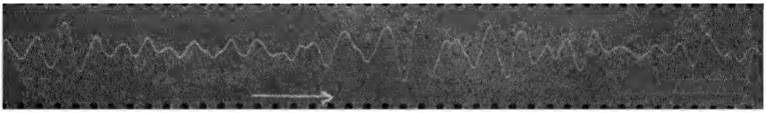

FIG. 149.—The Barkhausen effect due to tumbling 50 small pieces of lodestone inside a small coil attached to an amplifier.

78. *X*-ray Evidence Concerning Magnetization Processes.—Thus far all the experiments have indicated that something turned within the medium when it was magnetized. Exactly what that unit is, which does turn, is not made plain by experiment. Its architectural plan has not yet been definitely laid down on paper.

The experiments of the Comptons and their co-workers, however, have eliminated certain possibilities if their interpretations are correct. The first experiment by K. T. Compton and E. A. Trousdale[1] was an attempt to find a variation in the Laue figures of a crystal when it was magnetized. They found no change and concluded that magnetization did not shift the atoms sufficiently to change the general form of the space lattice in which they are arranged. Furthermore, they argued that the ultimate magnetic particle is not a group of atoms, such as the chemical molecule, but is the individual atom or something within the atom.

Later, A. H. Compton and O. Rognley[2] attacked the same problem by another *x*-ray method. Their point of view was that the relative intensity of the different orders of an *x*-ray spectrum

[1] COMPTON, K. T. and TROUSDALE, E. A., *Phys. Rev.*, **5**, 315, 1915.

[2] COMPTON, A. H., and ROGNLEY, O., *Science*, **46**, 415, 1917; *Phys. Rev*, **16**., 464, 1920.

line depends upon the distance of the electrons from the middle planes of the atomic layers in the diffracting crystal. In the unmagnetized condition the electronic orbits lie in helter-skelter fashion, so that on the average the electrons will be out an appreciable distance from the mid-planes of their atomic layers. Magnetic saturation, normal to the reflecting plane of a piece of magnetite, should turn all of the electronic orbits parallel to the reflecting surface. In this case the electrons are in the mid-planes of the layers of atoms which are effective in producing the reflected beam. This should give a difference in the relative intensity between the various orders of the x-ray spectra, as also would be the case when the surface is magnetized parallel to the reflecting surface. No effect was found and so they concluded that:

1. The ultimate magnetic particle is either the atom or something within the atom.
2. If the atom is the ultimate magnet, its electrons are not all distributed in the same plane, as assumed by Bohr, but are arranged very nearly isotropically.
3. Our experiments are in accordance with the hypotheses that the atomic nuclei or the electrons themselves are the ultimate magnetic particles.

These experimenters were inclined to consider the electron as the more probable elementary magnet, citing the experiments of Barnett and Stewart on gyromagnetic effects as supporting this point of view. It might be pointed out that the magneton proposed by Parsons[1] and the spinning electron suggested by King[2] emphasize the motion of negative electricity as the source of the elementary magnet.

More recently Yensen[3] has repeated the experiment of Compton and Rognley. Yensen's results indicate that sufficiently high magnetizing forces were used to insure saturation. The results are confirmatory of the previous work. If anything does turn within the medium, then these experiments of the Comptons are very important. They appear to be very conclusive that the something which turns is either the nucleus or the electrons in their orbits. They do not turn together.

[1] PARSON, *Smithsonian Misc. Coll.*, **65**, No. 11, November, 1915.
[2] KING, Privately printed.
[3] YENSEN, *Phys. Rev.*, **31**, 714, 1928.

79. Planetesimal Hypothesis of Magnetism.—In an electromagnet, the current in the coil and the iron core participate in the field which is produced. May not the nucleus of the atom and the orbital electrons cooperate in producing the elementary magnet? According to the Comptons' experiments the planes of the electronic orbits do not rotate. There is nothing in their experiments, however, to preclude the nucleus from turning. Such a point of view has been taken in the planetesimal hypothesis of magnetism.[1] It assumes that the nucleus of the atom is an oblate spheroid in the case of ferromagnetic substances. The tendency of this oblate spheroid is to set its equatorial plane either parallel or normal to the applied magnetic field, depending upon conditions. In the earlier concepts of this theory it was assumed that this nucleus turned, carrying with it the orbital electrons. The experiments of the Comptons seem to prohibit absolutely any such point of view. Adopting Ewing's last model, this nucleus becomes the Weber element and the same reasoning which Honda applied to his nuclear magneton may be used here also. The planes of the orbital electrons remain fixed in the medium. The negative electrons, as they revolve about the nucleus, induce a polarity in the nucleus. This polarized nucleus is then acted upon by the external magnetizing force. The idea of an oblate spheroid which could rotate has been introduced solely to account for the magnetostrictive and allied phenomena. Its introduction has not particularly modified the ordinary concepts of the regular magnetic phenomena except in a helpful way. The problems of hysteresis and residual magnetism may be treated just as Ewing has done for his model.

In the magnetostrictive effects there occur very definite mechanical effects due to a magnetic field. To explain the longitudinal and transverse Joule effect[2] demanded a mechanism such as the rotation of an oblate spheroid. To explain why the most rapid change in induction occurred at the same field strength as the most rapid change in length[3] needed a similar model. The change in magnetization due to tension may be visualized by means of these spheroids. The picture of a tension mechanically rotating a group of oblate spheroids[4] make the process seem very

[1] Williams, *Phys. Rev.*, **32,** 249, 1911; **34,** 40, 1912; **35,** 282, 1912; **2,** 241, 1913; *School Sci. and Math.*, **22,** 859, 1922.

[2] Williams, *Phys. Rev.*, **4,** 504, 1914.

[3] Williams, *Phys. Rev.*, **34,** 258, 1912.

[4] Williams, *School Sci. and Math.*, **22,** 859, 1922.

tangible. This is clearly shown in Fig. 150 where an oblate spheroid is stuck in a sheet of rubber dam *a*. As the rubber is uniformly stretched a torque is impressed on the spheroid *b*, and turns until its major axis is parallel to the stress *c*. The peculiar changes in resistance in ferromagnetic bodies,[1] due to a magnetic field, finds a *raison d'être* in a model of this type. This hypothesis is particularly illuminating for the untwisting of permanently twisted bars when magnetized longitudinally.[2]

Thus far, we have no information as to the exact manner in which these elementary magnets are oriented in an unmag-

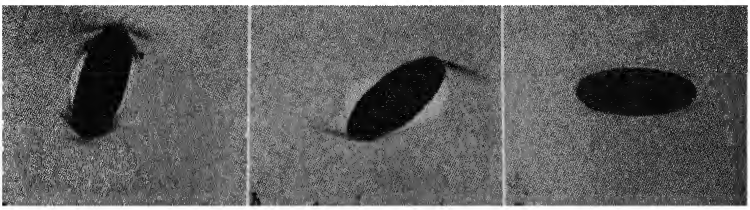

FIG. 150.—An oblate spheroid is placed in a hole cut in a sheet of rubber dam. Stretching of the rubber sheet causes the spheroid to turn with the major axis parallel to the direction of mechanical pull.

netized ferromagnetic body. We speak of random orientation but that does not necessarily mean that each elementary magnet is turned differently from its neighbor. The elementary units may act in groups composed of two, three, four, or more magnetons, but still be symmetrically arranged in the space lattice in which, we know, the metals exist. Such a point of view provides an explanation for the differences in magnetic properties along different axes of a crystal. A polycrystal specimen would give all the random orientation needed for ordinary magnetic properties. By means of various groupings[3] it seems possible to devise schemes which would give the different types of the Joule effect shown in Fig. 86. If something turns within the medium, x-ray examination does not furnish us with the clue as to how these units have their axes turned. The planetesimal model requires something to turn just as do the other theories.

80. Magneto-chemical Theories.—There has been going on, in recent years, a very decided tendency to invoke the aid of

[1] WILLIAMS, *Phys. Rev.*, **2**, 241, 1913.
[2] WILLIAMS, *Amer. Jour. Sci.*, **36**, 555, 1913.
[3] WILLIAMS, *Phys. Rev.*, **34**, 45, 1912.

magnetism in explaining some of the problems involved in chemical valence. That magnetism might influence chemical reaction has been a subject of study for a considerable length of time.[1] The extent to which some investigators have gone is given by the opening sentence in an address delivered by G. N. Lewis at the dedication of the Sterling Chemical Laboratory at Yale University. He says, "The electrochemical theory, which at several epochs has so largely dominated the science of chemistry, must now be abandoned or reduced to quite an insubordinate rôle." Lewis builds up a very plausible theory of valency, based on the tendency of electrons to occur in pairs. By occurring in pairs, he means that two electrons, revolving in orbits, behave as magnets. It is the magnetic attraction between these magnets which causes the electrons to pair off and gives very decided chemical differences between molecules with an odd number of electrons and those with even. Atoms with a magnetic moment have an odd number of electrons and are paramagnetic, while the great majority of even atoms possess no magnetic moment. The elements with an odd number of electrons should show in general the greater susceptibilities. This is strikingly shown in the curve[2] for the relation between atomic number and specific susceptibility.

81. Pascal's Work.—Pascal[3] measured the susceptibilities of a great many organic compounds, thus relating magnetic and chemical properties in a very intimate way. Out of his work came this rather important point, that diamagnetic atoms combine to give diamagnetic molecules, and that to a first approximation the effects are additive. Paramagnetic atoms, on the other hand, frequently lose their paramagnetic character on combination. Ferromagnetics generally form non-ferromagnetic compounds, but manganese, which is paramagnetic, shows a strong tendency to form compounds which are ferromagnetic. The Heusler[4] alloy is a ternary alloy illustrating this point. The alloy, *Al* 10 per cent, *Mn* 20 per cent, *Cu* 70 per cent, is the most

[1] REMSEN, *Amer. Chem. Jour.*, **3**, 157, 1881;
ROWLAND and BELL, *Phil. Mag.*, **26**, 105, 1888;
LOB, *Amer. Chem. Jour.*, **13**, 145, 1891;
DE HAUPTUNE, *Zeitsch. phys. Chem.*, **34**, 669, 1900;
PASCAL, *Nature*, 116, p. 923, 1925.

[2] ANDRADE, "The Structure of the Atom," p. 583, 1927.

[3] PASCAL, *Ann. chim. phys.*, **19**, 5, 1910; **25**, 289, 1912.

[4] HEUSLER, STARK and HAUPT, *Verh. d. phys. Gesellsch.*, **5**, 219, 1903.

highly magnetizable of all. Heusler believed the "carrier" of the ferromagnetism is a compound given by the formula, $Al_x(MnCu)3x$. Another hypothesis is that the manganese exists in the Heusler alloys as an allotropic form which is ferromagnetic. On this basis it might be expected that manganese would be ferromagnetic at low temperatures. K. Onnes and Weiss found that at 14° absolute, manganese was still paramagnetic. A third hypothesis builds on the crystalline structure of the alloy. This is only going back and asking what produces the property of ferromagnetism. In this connection it can only be pointed out that the magnetic properties, as related to crystalline structure, would fill volumes if it were all written up and, of course, plays its part in magneto-chemistry. Tyndall and others studied the behavior of many kinds of crystals in a magnetic field and referred to it as magnecrystallic action. This effect, as recently pointed out by Bragg,[1] is closely related to the problem of the effect of stress on the constitution and physical properties of materials.[2] Much work needs to be done in this field for we are still far from the full explanation of the connection between structure and magnetism, and of the influence of the latter on physical properties. We are still in our infancy as touching the field of magneto-chemistry.[3]

Even in the magneto-chemical theory of Lewis[4] there is a demand for the possibility of something being able to rotate in the medium, in order for the bonds between the atoms to be effective. No other hypothesis seems more strongly entrenched in our theories of magnetism than that of rotation of some element within the medium when it is magnetized. What that portion of matter is, which does turn, if it does, seems not so clear in our theories. Wave mechanics may offer some hope of a solution. It may mean that we should begin all over again with our theories. *The determination of what the elementary magnet is, constitutes the fundamental problem in magnetic research.*

[1] Bragg, *Nature*, **119**, 61, 1927.

[2] Joffe, "The Physics of Crystals," p. 12 *et seq.*, 1928.

[3] Wedekind, "Magnetochemie."

[4] Lewis, "Chemical Reviews," vol. I, p. 231, 1924; *Jour. Amer. Chem. Soc.*, **46**, 2027, 1924.

MAGNETIC DATA

The following tables are added for illustrative purposes only. No attempt has been made to make them comprehensive. The "International Critical Tables" cover the subject of magnetic properties so thoroughly that there is no point in duplication.

TABLE I.—DEMAGNETIZING FACTORS TO BE USED WITH THE EQUATIONS $H = H_0 - NI$ AND $H = H_0 - KB$

By C. L. B. SHUDDEMAGEN[1]

Values for N

m.	Method of steps		Method of reversals		
	D. = 0.32 cm. (1/8 in.)	D. = 0.6 to 2 cm. (1/4 to 3/4 in.)	D. = 0.32 cm. (1/8 in.)	D. = 0.4 cm. (5/32 in.)	D. = 0.6 to 2 cm. (1/4 to 3/4 in.)
10	0.220	0.204	0.215	0.210	0.200
15	0.117	0.106	0.113	0.1095	0.104
20	0.074	0.0672	0.0720	0.0690	0.0655
25	0.0515	0.0467	0.0503	0.0480	0.0455
30	0.0382	0.0344	0.0372	0.0356	0.0335
35	0.0295	0.0264	0.0279	0.0273	0.0258
40	0.0234	0.0211	0.0228	0.0217	0.0206
45	0.0191	0.0172	0.0186	0.0177	0.0167
50	0.0160	0.0144	0.0154	0.01475	0.0139
60	0.0115	0.0104	0.0112	0.0107	0.0101
70	0.0087	0.00795	0.0085	0.00816	0.0077
80	0.0069	0.00625	0.0067	0.00645	0.0061
90	0.0056	0.00507	0.0054	0.00521	0.00495
100	0.0046	0.00420	0.00445	0.00432	0.00410
125	0.0031	0.00280	0.00297	0.00282	0.00273
150	0.00222	0.00204	0.00214	0.00209	0.00198
175	0.00166	0.00154	0.00161	0.00158	0.00150
200	0.00130	0.00120	0.00125	0.00122	0.00117

[1] *Phys. Rev.*, **31**, 168, 1910.

TABLE I.—(*Continued*)

Values for K

m.	Method of steps		Method of reversals		
	D. = 0.32 cm. (1/8 in.)	D. = 0.6 to 2.0 cm. (1/4 to 3/4 in.)	D. = 0.32 cm. (1/8 in.)	D. = 0.4 cm. (5/32 in.)	D. = 0.6 to 2.0 cm. (1/4 to 3/4 in.)
10	0.0175	0.0162	0.0171	0.0167	0.0159
15	0.0093	0.0084	0.0090	0.0087	0.0083
20	0.0059	0.00535	0.0057	0.0055	0.0052
25	0.0041	0.00372	0.0040	0.00382	0.00362
30	0.00304	0.00274	0.00296	0.00283	0.00267
35	0.00235	0.00210	0.00222	0.00217	0.00205
40	0.00186	0.00168	0.00181	0.00173	0.00164
45	0.00152	0.00137	0.00148	0.00141	0.00133
50	0.00127	0.00115	0.00122	0.00117	0.00110
60	0.00091	0.00083	0.00089	0.00086	0.00080
70	0.00069	0.00063	0.00067	0.00065	0.000613
80	0.00055	0.00050	0.00053	0.00051	0.000485
90	0.000445	0.000404	0.00043	0.000415	0.000394
100	0.000366	0.000334	0.000354	0.000344	0.000326
125	0.000247	0.000223	0.000236	0.000224	0.000217
150	0.000177	0.000162	0.000170	0.000166	0.000158
175	0.000132	0.000122	0.000128	0.000126	0.000119
200	0.000103	0.000096	0.000100	0.000097	0.000093

TABLE II.—NORMAL MAGNETIC INDUCTION

Electrolytic iron fused in vacuum and annealed; C = 0.0015 per cent

H	B_n
0.2	6,700
0.4	12,900
1 0	14,600
4.0	15,400
20.0	16,400
100.0	18,400

Annealed Swedish charcoal iron. Annealed at 900° C.

H	μ
0.25	1,240.0
0.5	2,000.0
0.75	4,530.0
1.0	6,350.0
1.5	5,600.0
2.5	4,220.0
5.0	2,590.0
10.0	1,460.0
20.0	810.0
50.0	340.0
100.0	181.0
150.0	125.7
300.0	67.3
500.0	42.3
1,000.0	22.04
2,000.0	11.57
3,000.0	8.06
4,500.0	5.71

Eutectoid steel. Oil-quenched at 800° C. and undrawn. 0.05 per cent *Cr* at 20° C.

H	B_n
20	700
50	3,250
100	8,950
150	11,380
200	12,750
300	14,050
500	15,550
700	16,500
1,000	17,450
1,500	18,580
2,000	19,350
2,500	20,000

TABLE II.—(*Continued*)

Soft cobalt. At 20° C.

H	$\mathfrak{I}_n$
5.1	72
10.7	170
13.9	247
22.8	371
35.7	497
54.5	628
85.9	749
147.7	865
244.3	953
392.0	1,027
476.0	1,047
610.0	1,088

Nickel: Nickel = 99.15 per cent; Mn = 0.7 per cent.

H	B_n	n
0.25	100	400.0
0.5	230	460.0
0.75	480	530.0
1.0	650	650.0
1.5	1,350	900.0
2.5	2,800	1,120.0
5.0	4,330	865.0
10.0	4,940	495.0
20.0	5,400	270.0
50.0	5,850	117.0
100.0	6,200	62.0
150.0	6,400	43.0
300.0	6,700	22.0
500.0	6,910	14.0
1,000.0	7,370	7.4
2,000.0	8,400	4.2
3,000.0	9,380	3.1
4,000.0	10,400	2.6

TABLE III.—MAGNETIC SUSCEPTIBILITIES

Gases

Unit of K = 10^{-10} cgsm. $\frac{K}{\rho} = \chi$

Element	t = 20° C. and p = 76 cm. Volume susceptibility	Source of data
H_2	-1.70×10^{-10}	WILLS and HECTOR
O_2	10.62×10^{-10}	
N_2	-4.91×10^{-10}	
He	-0.78×10^{-10}	
Ne	-2.77×10^{-10}	
A	-7.52×10^{-10}	

Liquids

Unit of χ = 10^{-6} cgsm.
t = 20° C.

Substance	Specific susceptibility	Source of data
Hg	-0.19×10^{-6}	"International Critical Tables"
H_2SO_4	-0.441×10^{-6}	
HNO^3	-0.467×10^{-6}	
HCl	-0.661×10^{-6}	
H_2O	-0.72×10^{-6}	
$NiSO_4$	27.10×10^{-6}	

Solids

Element	Specific susceptibility	Source of data
Cu	-0.086×10^{-6}	"International Critical Tables"
Pb	-0.12×10^{-6}	
Ag	-0.20×10^{-6}	
Al	0.63×10^{-6}	
Pt	1.10×10^{-6}	
U	2.6×10^{-6}	

TABLE IV.—MAGNETOSTRICTION

Longitudinal Joule Effect

Tabular values $= A$. $A \times 10^{-6} = \frac{l - l_0}{l_0}$, where l_0 = length when $H = 0$

H	Soft iron 8.4° C. A	Nickel 17.2° C. A	Huesler alloy 20° C. A
50	1.60	−9.60	0.990
100	3.20	−20.00	1.270
150	3.03	−26.00	1.405
200	2.40	−29.70	1.505
300	0.55	−33.10	1.627
400	−1.15	−34.70	1.692
500	−2.63	−35.45	1.723
600	−3.90	−36.03	
700	−4.95	−36.50	
800	−5.70	−36.82	
900	−6.05	−37.10	

TABLE V.—CHANGE IN RESISTANCE DUE TO MAGNETIC FIELDS

Wire of electrolytic bismuth at 18° C. in a transverse field. Tabular values $A = \frac{r - r_0}{r_0}$, where r_0 = resistance when $H = 0$, $\lambda = 0.5893\mu$

H	A
4	0.12
8	0.32
12	0.52
16	0.79
20	1.04
25	1.33
30	1.65
35	2.01

Nickel at 18° C.

Unit of $\frac{\Delta r}{r} = 0.0001A$

Transverse field		Longitudinal field	
H	A	H	A
1	7	0.15	60
2	3	0.30	98
3	−36	0.48	120
6	−83	0.60	126
10	−95	0.90	129
14	−104	1.20	130
20	−117	1.50	131
25	−129	1.80	131
35	−150		

TABLE VI.—THE FARADAY EFFECT

Verdet's Constant for $\lambda = 0.5893$

Gases

Unit of $V = 10^{-6}$ minutes of arc per centimeter gauss. $p = 10^6$ barye = $0.987 A_n$

Substance	Constant, V	Pressure	Temperature, ° C.
H	456.00	83.30	9.5
N	549.00	98.10	14.0
O	555.00	98.10	7.0
CS_2	23.49	0.98	70.0
Air	551.00	98.10	13.0
CO_2	8.61	1.10	65.0

Liquids

Unit of V = 0.001 minute of arc per centimeter gauss

Substance	Constant, V	Temperature, ° C.
H_2O	13.08	20
H_2SO_4	10.50	16
HCl	11.60	20
HNO_3	8.76	16
CS_2	43.00	18
C_2H_5OH	11.12	25

Solids

Unit of V = 0.001 minutes of arc per centimeter gauss

Substance	Constant, V	Temperature, °C.
Amber	−9.60	19
Glass, Jena, S.179	16.10	18
Glass, Jena, 0.451	31.70	18
Glass, Jena, S.143	88.80	18
NaCl	38.85	16
KCl (sylvite)	28.58	16

TABLE VII.—KERR EFFECT

Unit of K_λ = 0.001 minutes of arc per cgsm. unit of wave length = 0.530μ

Substance	H	K_λ
Electrolytic iron	40	−13.5
Electrolytic cobalt	34	−14.7
Pure nickel	20	−15.7

TABLE VIII.—MAGNETIC ELEMENTS AT VARIOUS PLACES ON THE EARTH'S SURFACE

Unit of H and V = 1γ = 10^{-5} gauss = 10^{-5} cgsm.

Place	Declination	Inclination	Horizontal component	Vertical component
Cheltenham	6° 27.7′ W.	70° 57.6′ N.	19020	55115
Greenwich	13° 35.1′ W.	66° 51.9′ N.	18431	43137
Odessa	3° 35.9′ W.	62° 26.9′ N.	21707	41606
Potsdam	6° 56.9′ W.	66° 32.8′ N.	18614	42905
Tiflis	3° 09.1′ E.	56° 51.1′ N.	25217	37612
Batavia-Buitenzorg	0° 52.9′ E.	32° 04.3′ S.	36821	23072
Apia	10° 19.2′ E.	30° 07.5′ S.	35249	20453
Watheroo	4° 18.3′ W.	64° 05 2′ S.	24750	50941

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